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Revisiting the box counting algorithm for the correlation dimension analysis of hyperchaotic time series

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We undertake the correlation dimension analysis of hyperchaotic time series using the box counting algorithm. We show that the conventional box counting scheme is inadequate for the accurate computation of correlation dimension (D_2) of a hyperchaotic attractor and propose a modified scheme which is automated and gives better convergence of D_2 with respect to the number of data points. The scheme is first tested using the time series from standard chaotic systems, pure noise and data added with noise. It is then applied on the time series from three standard hyperchaotic systems for computing D_2. Our analysis clearly reveals that a second scaling region appears at lower values of box size as the system makes a transition into the hyperchaotic phase. This, in turn, suggests that correlation dimension analysis can also give information regarding chaos-hyperchaos transition.
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1. Introduction

Since the introduction of the Lorenz system in 1963 [1], the developments in the field of deterministic chaos have been very rapid. The Lorenz attractor revealed all the complex and fundamental features of a typical low dimensional chaotic attractor and was instrumental in the development of many new mathematical and numerical techniques for the analysis of dynamical systems in general. Along with the theoretical advances, a large number of practical applications have also been developed for chaotic systems over the past four decades, on account of their complex dynamics and the self similar fractal structure of the underlying attractors.

But recently, with regard to practical applications, much of the attention has been shifted to systems producing hyperchaotic attractors. They are characterised by more than one positive Lyapunov exponents (LE) and are much more complex in terms of topological structure as well as dynamics compared to low dimensional chaotic attractors. Hence hyperchaotic attractors are preferred for many of the applications that require the complexity of the dynamics, as in network security [2] and data encription [3,4] and also in many synchronisation studies using electro-optic devices [5,6]. In fact, hyperchaos was first reported by Rossler in 1979 [7], but became popular only in the last decade or so when its potential in secure communication was realised by various scientific and engineering communities.

Broadly, there are two types of nonlinear dynamical systems that can generate hyperchaotic attractors. One is a system of coupled autonomous differential equations (flow) or a system of coupled oscillators and the second is a nonlinear time delayed differential equation. In the former case, the dimensionality of the system should be at least four to display hyperchaotic behavior since there are at least two directions of stretching. A large number of such systems have been

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developed and analysed in the last few years [8–10]. The hyperchaotic flows are widely used in data encription and secure messaging.

The most important examples of the second kind are the Mackey–Glass (M-G) system [11] and the Ikeda system [12]. Here the behavior of the system crucially depends on the time delay parameter, usually represented by τ. Depending on the value of τ, the system displays a range of periodic, chaotic as well as hyperchaotic dynamics. These systems are mainly developed from a biological perspective and more details regarding these systems are discussed in §4.

Hyperchaotic systems are mainly analysed by computing the LEs as a function of the control parameters. An important aspect in their quantitative analysis is the transition from chaotic to hyperchaotic phase. Generally, this occurs when one of the control parameters passes through a critical value, when the second largest LE turns positive. When the equations governing the time evolution of the dynamical system are known, the transition from chaos to hyperchaos can be readily obtained through LE analysis. But, if the only information available on the system is a time series, such a method is more difficult to apply. In such cases, one is more inclined to use other measures, such as, D_2 or recurrence plots [13].

As the attractor becomes more complex with two directions of stretching in the hyperchaotic phase, the fractal dimension can also vary in a nontrivial manner. But the dimensional analysis of hyperchaotic systems have been very few in the literature. The main reason for this could be the practical difficulty for computing the dimension from a hyperchaotic time series, as discussed in detail in the next section. But in one of the attempts, [14], Kapitaniak et al. [15] have shown that D_2 displays more than one scaling region in the hyperchaotic phase for a system of unidirectionally coupled oscillators. The authors further show that the transition to hyperchaos is mediated by changes in the stability of an infinite number of unstable periodic orbits embedded in the chaotic attractor whose basin becomes riddled [16]. Thus, computing D_2 can give useful information regarding chaos-hyperchaos transition. Moreover, D_2 is an important quantifier that can represent the geometric complexity of a hyperchaotic attractor, which is vital for many practical applications of hyperchaos.

The computation of D_2 is usually done using a time series employing the delay embedding technique. Several methods have been proposed in the literature over the past two decades for computing D_2 from a time series, such as, the Grassberger–Procaccia (GP) algorithm [17], the box counting algorithm [18], the box assisted correlation algorithm by Theiler [19], the false nearest neighbour method [20], the gaussian kernal method by Diks [21] and the algorithm by Judd [22], to name a few. But none of the above methods have specifically adressed the issue of computing D_2 from a hyperchaotic time series, which is our main concern in this paper. Among the methods mentioned above, the GP algorithm and the box counting algorithm have gained more popularity due to their wide applicability. Hence we consider these two methods, to see which one is more suitable for the analysis of hyperchaotic time series.

We have recently proposed and implemented a non subjective approach to the GP algorithm [23] to compute D_2, that can be applied to synthetic as well as real world data involving noise. But the GP algorithm involves computation of distances from a reference point for calculating the correlation sum. As a consequence, it is much more expensive in terms of computer time and memory compared to the box counting algorithm, where the computer mainly does comparison and sorting. This is especially true if the number of data points in the time series is required to be large, as in the case of hyperchaotic time series. Moreover, since D_2 is typically large for these systems, the computation has to proceed to a much higher embedding dimension to ensure a reasonable saturation of D_2. The above factors give a defnite advantage for the box counting algorithm over the GP algorithm in the analysis of hyperchaotic time series, which motivate us to undertake a systematic study of the box counting scheme.

We then find that the conventional box counting scheme is inadequate for the D_2 analysis of hyperchaotic time series, as shown in detail in the next section. Here we propose a modified scheme by redefining the expression for probability for computing D_2. By doing this, we are able to push the scaling region to much smaller values of box size compared to the conventional scheme. We show that this gives excellent results for hyperchaotic systems and we also bring out some interesting features of the hyperchaotic attractor using the modified scheme.

Our paper is organised as follows: In the next section, we point out the main drawbacks of the conventional box counting scheme and propose a modified scheme to compute D_2. In §3, the scheme is tested using several standard chaotic time series, random data and data added with noise. The scheme is then applied to the D_2 analysis of hyperchaotic systems in §4. Finally, the conclusions are drawn in §5.

2. Revisiting the box counting algorithm

2.1. The conventional scheme

The box counting algorithm for computing D_2 from a time series has been discussed by many authors [18,24–26] in the past. To apply the box counting scheme, one first constructs a delay embedding attractor from the time series in an embedding space of dimension M with a suitably chosen time delay τ. One then tries to cover the attractor using M dimensional cubes of side length r. The correlation dimension D_2 is then defined by the expression

$$D_2 \equiv \lim_{r \to 0} \log C(r) / \log(r), \tag{1}$$

with $C(r)$ given by

$$C(r) = \sum_{i=1}^{N(r)} p_i^2(r). \tag{2}$$

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