Commun Nonlinear Sci Numer Simulat 19 (2014) 2919–2927

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Short communication

Chaos in a new fractional-order system without equilibrium points

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ARTICLE INFO

Article history:
Received 18 June 2013
Received in revised form 14 February 2014
Accepted 15 February 2014
Available online 26 February 2014

Keywords:
Chaotic dynamics
Fractional calculus
Equilibrium points
Predictor–corrector algorithm
Lyapunov exponents

ABSTRACT

Chaotic systems without equilibrium points represent an almost unexplored field of research, since they can have neither homoclinic nor heteroclinic orbits and the Shilnikov method cannot be used to demonstrate the presence of chaos. In this paper a new fractional-order chaotic system with no equilibrium points is presented. The proposed system can be considered "elegant" in the sense given by Sprott, since the corresponding system equations contain very few terms and the system parameters have a minimum of digits. When the system order is as low as 2.94, the dynamic behavior is analyzed using the predictor–corrector algorithm and the presence of chaos in the absence of equilibria is validated by applying three different methods. Finally, an example of observer-based synchronization applied to the proposed chaotic fractional-order system is illustrated.

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1. Introduction

Fractional calculus is a mathematical topic more than 300 years old, but its application to physics and engineering has been developed only in recent years [1]. This happens because it has been recently found that several physical phenomena can be more accurately described by fractional differential equations rather than integer-order models [2]. A number of techniques are available for approximating fractional derivatives and integrals [3–5]. Therefore, at present fractional calculus plays an important role in physics [2], electrical circuit theory [6] and control systems [7]. In particular, a significant role is played in chaos theory, where it has been shown that chaotic phenomena can be obtained in nonlinear dynamic systems with fractional-order [8,9]. To this purpose, several chaotic fractional-order systems have been proposed starting from the chaotic integer-order counterparts. For example, by considering the pioneering Chua's circuit [10], some fractional order counterparts have been proposed [11–13]. Similarly, by considering the well-known integer-order Chen system [14], a number of fractional order systems have been studied [15,16]. Moreover, the fractional Rössler system [17] and the fractional multi-scroll system [18] have been developed starting from the corresponding integer-order systems [19,20]. Note that all the previous chaotic integer-order systems, along with their chaotic fractional-order counterpart, are characterized by one or more equilibrium points. On the other hand, very few papers have focused on the study of chaotic dynamics in integer-order and fractional-order systems without equilibria.

Referring to integer-order systems with no equilibria, the presence of chaos has been investigated only in [21–23]. In particular, in [21] a systematic search to find 3D integer-order chaotic systems with quadratic nonlinearities and no equilibria was performed. The objective was to find the algebraically simplest cases which cannot be further reduced by the removal of

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http://dx.doi.org/10.1016/j.cnsns.2014.02.017
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terms without destroying the chaos. Note that the presence of chaos in these systems is very surprising since they can have neither homoclinic nor heteroclinic orbits [24], and thus the Shilnikov method [24] cannot be used to verify the chaos. Referring to fractional-order systems, the study of chaotic systems with no equilibria is an almost unexplored topic, given that only one paper [25] has been published in literature to date.

Based on these considerations, in this paper a new fractional-order chaotic system with no equilibrium points is presented. The proposed system, which represents the fractional-order counterpart of an integer-order system without equilibria studied in [21], can be considered "elegant" in the sense of Sprott [26] since the corresponding system equations contain no unnecessary terms and the system parameters have a minimum of digits. The presence of fractional chaos in the absence of equilibria has been validated by applying three different numerical tests when the system order is as low as 2.94.

The paper is organized as follows. In Section 2 the fundamentals of fractional derivative and the predictor–corrector method for solving fractional-order equations are illustrated. In Section 3 the equations of the considered fractional-order system with no equilibria are given, whereas in Section 4 the predictor–corrector algorithm is applied to solve the system. In particular, a chaotic attractor is found when the order of the derivative is $q = 0.98$. Moreover, three different numerical methods, including the recently introduced "0–1 test" for chaos [27], are applied to further confirm the presence of chaos in the proposed fractional-order chaotic system without equilibrium points. Finally, an example of observer-based synchronization applied to the considered chaotic fractional-order system is provided.

2. Theoretical background

The *Riemann–Liouville fractional integral operator* $J_{t_0}^q$ of order $q \in \Re^+$ is defined on the Lebesque space $L_1[t_0, t_1]$ by

$$J_{t_0}^q x(t) \triangleq \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-\tau)^{q-1} x(\tau) d\tau, \tag{1}$$

where $\Gamma(q)$ is the Gamma function, with $J_{t_0}^0 x(t) = x(t)$ [28].

For $\gamma > -1$ and C a real constant, two fundamental properties of the integral operator $J_{t_0}^q$ are [28]:

$$J_{t_0}^q (t-t_0)^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1+q)} (t-t_0)^{\gamma+q}; \tag{2}$$

$$J_{t_0}^q C = C \frac{\Gamma(1)}{\Gamma(q+1)} (t-t_0)^q = \frac{C}{\Gamma(q+1)} (t-t_0)^q. \tag{3}$$

Referring to the fractional differential operators, among the different definitions proposed in the literature, in this work the differential operator ${}^*D_{t_0}^q$ proposed by Caputo is utilized:

$${}^*D_{t_0}^q x(t) \triangleq J_{t_0}^{m-q} D_{t_0}^m x(t) = \frac{1}{\Gamma(m-q)} \int_{t_0}^{t} \frac{x^{(m)}(\tau)}{(t-\tau)^{q+1-m}} d\tau. \tag{4}$$

where $m-1 < q \leqslant m$ and $m \in N$ (i.e., $m = \mathrm{ceil}(q)$) [29]. Two fundamental properties of the differential operator ${}^*D_{t_0}^q$ are [28]:

$${}^*D_{t_0}^q J_{t_0}^q x(t) = x(t); \tag{5}$$

$$J_{t_0}^q ({}^*D_{t_0}^q) x(t) = J_{t_0}^q J_{t_0}^{m-q} D_{t_0}^m x(t) = J_{t_0}^m D_{t_0}^m x(t) = x(t) - \sum_{k=0}^{m-1} x^{(k)}(t_0^+) \frac{(t-t_0)^k}{k!}. \tag{6}$$

Based on the Caputo's definition (4), consider the following general form of fractional-order differential equation:

$${}^*D_{0,t}^q x(t) = f(x(t)), \quad x(0) = x_0, \quad q \in (0,1). \tag{7}$$

The initial value problem (7) is equivalent to a Volterra integral equation [30],

$$x(t) = x_0 + \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau, x(\tau)) d\tau \tag{8}$$

in the sense that a continuous function solves (8) if and only if it solves (7). Now, the predictor–corrector algorithm, which belongs to the family of Adams–Bashforth–Moulton schemes, is adopted to solve the Volterra integral equation (8). Firstly, the product trapezoidal quadrature formula is applied to replace the integral in Eq. (8). By taking $0 \leqslant t \leqslant T$ and by setting $h = T/N$ ($N \in \mathbb{Z}^+$), $t_n = nh$, $n = 0, 1, \ldots, N$, Eq. (8) can be discretized as [30]:

$$x(t_{n+1}) = x_0 + \frac{h^q}{\Gamma(q+2)} \sum_{j=0}^{n+1} \alpha_{j,n+1} f(t_j, x(t_j)), \tag{9}$$

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