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Existence and global stability of a periodic solution for a cellular neural network ☆

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ABSTRACT

Purpose of this study is to investigate the dynamical properties of Chua and Yang cellular neural networks (CNNs). Based on the continuation theorem of coincidence degree theory, a novel sufficient condition with respect to the existence of periodic solution for CNNs is derived. Moreover, a generalized Lyapunov–Krasovskii functional is designed to guarantee the global stability of the existed periodic solution. An illustrative example is given to verify the effectiveness and correctness of the proposed method, furthermore, random disturbance is added in the numerical simulation in order to verify the robustness of the proposed approach.

1. Introduction

CNN is defined as an analog parallel computing paradigm in space, i.e. image processing tasks [1,2], partial differential equations (PDE) systems [3,4], signal processing, associative memories [5], pattern classification [6,7], and characterized by locality of connections between processing elements (cells or neurons) [8]. Such applications rely on the existence of equilibrium points or a unique equilibrium point and qualitative properties of stability. In [9,10], CNNs with multiple time delays with impulsive effects and nonlinear chaotic dynamic behaviors are, respectively investigated. Sufficient conditions for exponential convergence characteristics of continuous-time CNNs with discrete independent delays are studied by utilizing Lyapunov functional associated with the constant input sources are formulated in [11]. Some models about CNNs have received increasing interest because their impressive applications in areas such as classification, parallel computing, associative memory, pattern recognition, computer vision, and solving some optimization problem [12–16]. The sufficient conditions related to the synchronization of the coupled identical Yang–Yang type fuzzy CNNs with time-varying delays are proposed in [17] based on the simple adaptive controller. Moreover, the synchronization of delayed fuzzy CNNs with impulses and unknown parameters are studied in [18], and the stability criteria for the system with impulses are derived

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by Lyapunov–Lasall principle. Li et al. [19] utilized the CNNs and linear matrix inequality (LMI) techniques for edge detection of noisy images, and the training template of noise reduction and edge detection CNNs are focused and discussed. In terms of the LMI criteria, the issue related to the existence of a unique equilibrium point and delay-dependent global asymptotical stability for CNNs with time-delay is addressed in [20]. Ref. [21] explores different alternatives to carry out a model refer to digital CNN from its implementation on Field-Programmable Gate Array (FPGA), and each given discrete approach is simulated and compared with the approaches results of continuous models. The hardware implementation of non-linear multi-layer CNN is described in [22], and proposed results can be used to process standard video in real time. A novel sufficient condition is given by Shao et al. [23] to obtain the discrete-time analogues of CNN with periodic coefficients in the three-dimensional space, and existence and global stability of a periodic solution for the discrete-time cellular neural network (DTCNN) are analyzed by utilizing continuation theorem of coincidence degree theory and Lyapunov stability theory, respectively. The binary input and output DT-CNNs associated with suitable templates are employed to solve the image-processing problem, connected component detection (CCD) by Chen et al. [24], and DTCNNs-based image-processing chip is implemented based on FPGA technology. Civalleri et al. [25] figures out that if delay of actual networks with periodic cycle is suitably chosen, CNNs with delay though symmetric may become unstable, it presents a sufficient condition ensure complete stability based on relationship between delay time and parameters. Moreover, the further sufficient condition related to the existence of a globally asymptotically stable equilibrium point in CNN's with dominant nonlinear delay-type templates is formulated in [26]. In [27], the feedback and delayed feedback matrices are used to ensure the uniqueness and global asymptotic stability of the equilibrium point for delayed CNNs. Furthermore, the [28] has proposed a new methodology for real-time processing of DNA chip images and the corresponding developed method is to use CNN array to analyze DNA microarray. A comprehensive review literature [29] for CNNs discussed the potential industry applications associated to cellular nonlinear/neural network technology, and it provided many examples and exercises, including CNN simulator, development software accessible and visual computing.

The paper is organized as follows: In Section 2, CNNs model and preliminaries are stated and some definitions and lemmas are listed. In Section 3, based on the continuation theorem of coincidence degree theory, a novel sufficient condition with respect to the existence of periodic solution for the CNN is derived. The global stability of a periodic solution of CNNs is addressed by constructing a Lyapunov function in Section 4. An illustrative numerical example is presented to verify the correctness of the proposed theorems in Section 5 and we concluded in Section 6.

2. Preliminaries and CNNs model

2.1. Preliminaries

Definition 1 (*Fredholm operator*). Let $\mathscr{X}$ and $\mathscr{Y}$ be a Banach space, an operator L is called Fredholm operator if L is a bounded linear operator between $\mathscr{X}$ and $\mathscr{Y}$ whose kernel and cokernel are finite-dimensional and whose range is closed.

Equivalently, an operator $\mathscr{L}:\mathscr{X}\rightarrow\mathscr{Y}$ is Fredholm if it is invertible modulo compact operator, i.e., if there exists a bounded linear operator $S:\mathscr{Y}\rightarrow\mathscr{X}$ such that $Id_{\mathscr{X}}-SL, Id_{\mathscr{Y}}-LS$ are compact operators on $\mathscr{X}$ and $\mathscr{Y}$, respectively, where $Id_{\mathscr{X}}$ and $Id_{\mathscr{Y}}$ are the identity operator.

Definition 2 (*L-compact*). Operator N will be called L-compact on $\bar{\Omega}$ if open bounded set $QN(\bar{\Omega})$ is bounded and $K_p(I-Q)N:\bar{\Omega}\rightarrow\mathscr{X}$ is compact, where K_p is the inverse operator of N. Since ImQ is isomorphic to Ker$L$, there exists an isomorphism $J:\mathrm{Im}\,Q\rightarrow\mathrm{Ker}\,L$. The index of a Fredholm operator is $\mathrm{ind}\,L=\dim\ker L-\mathrm{codim}\,\mathrm{ran}\,L$ or equivalently $\mathrm{ind}\,L=\dim\mathrm{Ker}\,L-\mathrm{codim}\,\mathrm{Im}\,L$, then operator L will be called a Fredholm operator of index zero if $\dim\mathrm{Ker}L=\mathrm{codim}\,\mathrm{Im}L<+\infty$ and ImL is closed in $\mathscr{Y}$, and if we define a abstract Eq. (1) in Banach space $\mathscr{X}$,

$$Lx=\lambda Nx. \tag{1}$$

$L:\mathrm{Dom}\,L\subset\mathscr{X}\rightarrow\mathscr{Y}$ be linear operator, and $N:\mathscr{X}\rightarrow\mathscr{Y}$ be a continuous operator. If L is a Fredholm operator of index zero, there must exist continuous projectors $P:\mathscr{X}\rightarrow\mathscr{X}$ and $Q:\mathscr{Y}\rightarrow\mathscr{Y}$ such that

$$P:\mathscr{X}\cap\mathrm{Dom}\,L\rightarrow\mathrm{Ker}\,L,\quad \mathrm{Ker}\,L=\mathrm{Im}\,P;\quad Q:\mathscr{Y}\rightarrow\mathscr{Y}/\mathrm{Im}\,L,\quad \mathrm{Im}\,L=\mathrm{Ker}\,Q.$$

Equivalently, $L:\mathrm{Dom}\,L\cap\mathrm{Ker}\,P\rightarrow\mathrm{Im}\,L$ is invertible, and we denote the inverse of that operator by K_p.

Lemma 1 (*Gaines and Mawhin, 1979*). *Let $\mathscr{X}$ be a Banach space, L be a Fredholm operator of index zero and let $N:\bar{\Omega}\rightarrow\mathscr{X}$ be L-compact on $\bar{\Omega}$, $\Omega\subset\mathscr{X}$, where Ω is an open bounded set, suppose*:

(*i*) $Lx\neq\lambda Nx$, *for any* $(x,\lambda)\in(\partial\Omega\cap\mathrm{Dom}\,L)\times(0,1)$;
(*ii*) $QNx\neq 0$, *for any* $x\in\partial\Omega\cap\mathrm{Ker}\,L$;
(*iii*) $\deg(JQN,\Omega\cap\mathrm{Ker}\,L,0)\neq 0$.

Then $Lx = Nx$ has at least one solution in $\mathrm{Dom}\,L\cap\Omega$.