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Analysis of diffusion process in fractured reservoirs using fractional derivative approach

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ABSTRACT

The fractal geometry is used to model of a naturally fractured reservoir and the concept of fractional derivative is applied to the diffusion equation to incorporate the history of fluid flow in naturally fractured reservoirs. The resulting fractally fractional diffusion (FFD) equation is solved analytically in the Laplace space for three outer boundary conditions. The analytical solutions are used to analyze the response of a naturally fractured reservoir considering the anomalous behavior of oil production. Several synthetic examples are provided to illustrate the methodology proposed in this work and to explain the diffusion process in fractally fractured systems.

1. Introduction

It is well known that the flow distribution and propagation within naturally fractured reservoirs (double porosity systems) is controlled mostly by the distribution of fractures [1]. The presence of fractures at different scales represents an element of uncertainty and heterogeneity in the construction of a reservoir model [1,2]. Due to this reason, classical Euclidean models cannot describe the complexities of such systems. Alternatively, fractal theory provides a powerful method to describe the complex network of fractures [1–4].

Chang and Yortsos [5] were the first authors adopting a fractal model to describe the response of a double porosity system (naturally fractured reservoir). Acuña et al. [6] applied this model to naturally fractured reservoirs and found that the wellbore pressure is a power-law function of time. Flamenco-Lopez and Camacho-Velazquez [7] discussed two approaches to describe and analyze a naturally fractured reservoir with fractal geometry: (1) analysis of both transient and pseudo-steady-state flow periods of well pressure tests, (2) determination of fractal-model parameters from porosity well logs or from another source [1,8].

From another point of view, fractional calculus (FC) as the generalization of the ordinary calculus, attracted the attention of scientists from many disciplines. Capturing the memory of a dynamical phenomenon via integer-order calculus poses difficulties, while FC by introducing a kernel considers inherently the hereditary of complex processes. Specifically, in the last two decade petroleum engineering has been impressed by FC. Several authors used the concept of FC to describe the complexity of diffusion process in porous media [9–11]. These researchers developed a mathematical model to define the response of oil reservoirs.

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Nomenclature

A_{wb} wellbore area, ft^3 (L^3)
aV_s fractal parameter related to the porosity of the fracture network, L_{mf}^{3-d}
B oil formation volume factor, RB/STB (L^3/L^3)
c compressibility, psi^{-1} ($L\,t^2/m$)
C_s wellbore storage coefficient, bbl/psi ($L^4\,t^2/m$)
C_D dimensionless wellbore storage coefficient
d Euclidean dimension
d_{mf} mass fractal dimension
g acceleration of gravity, ft/s^2 (L/t^2)
g_c gravitational units conversion factor, 32.17 (lb_m/ft)/($lb_f\,s^2$)
h formation thickness, ft (L)
I modified Bessel function of the first kind
k permeability, md (L^2)
K modified Bessel function of the second kind
L Laplace transform of
m fracture-network parameter, $L^{2+\theta}$
p pressure, psi ($m/(Lt^2)$)
p_{Dw} dimensionless wellbore pressure for the constant-rate case without wellbore storage and skin effects
p_{wD} dimensionless wellbore pressure with wellbore storage and skin effects
p_{wf} wellbore flowing pressure, psi ($m/(Lt^2)$)
q production oil rate, STB/D (L^3/t)
q_D dimensionless rate
r radial distance, ft (L)
r_D dimensionless radial distance
S total skin factor
t time, hours (t)
t_D dimensionless time
u Laplace transform variable
γ fractional derivative order
Γ gamma function
θ conductivity index
λ interporosity flow parameter
μ oil viscosity, cp ($m/(Lt)$)
ρ density of liquid in the wellbore, lb_m/ft^3 (m/L^3)
σ matrix/fracture interaction index
ϕ porosity
ω storativity ratio

Subscripts
D dimensionless
e external
f fracture (fissure)
m matrix
w wellbore

It is generally accepted that the diffusion process is history-dependent in fractal systems. In fractally fractured reservoirs, history of flow and nonlocality are pivotal in all stages of production. Metzler et al. [9] applied a fractional derivative approach to fractal model to incorporate the memory.

To characterize a naturally fractured reservoir, it is required the estimation of the fractal and fractional parameters (dimensionless parameters) of the mathematical model. In spite of all the work done on fractally fractional diffusion (FFD), an appropriate analytical solution has not been addressed in the literature to calculate the fractal and fractional parameters and to analyze the effects of these parameters upon reservoir behavior. This work presents an analytical solution for the pressure response of naturally fractured reservoirs during transient and boundary-dominated flow periods, in the light of FC. In particular, the response of fractured systems is investigated on the basis of fractional derivative. The analytical solutions are discussed by several synthetic examples.

This paper is organized as follows. Section 2 introduces the fundamentals of fractional calculus. Sections 3 and 4 develop and introduce the FFD model and some synthetic examples, respectively. Finally, the concluding remarks are discussed in Section 5.