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Second-order cluster consensus of multi-agent dynamical systems with impulsive effects

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ABSTRACT

This paper discuss the cluster consensus of multi-agent dynamical systems (MADSs) with impulsive effects and coupling delays. Some sufficient conditions that guarantee cluster consensus in MADS are derived. In each cluster, agents update their position and velocity states according to a leader's instantaneous information, and interactions among agents are uncertain. Furthermore, switching topology problem in MADS is considered by impulsive stability and adaptive strategy. Finally, numerical simulations are given to verify our theoretical analysis.

1. Introduction

Recently, multi-agent dynamical systems have received major attentions due to growing interests in studying their collective behaviors, such as consensus, swarming, flocking, sensor networks and intelligent robots, etc. Many literatures [1–11] focused on various conditions that ensure all autonomous mobile agents achieve a certain global behavior of common interest. For example, relation between consensus and interaction graph of MADS was discussed in Refs. [1,2]. Controllability and connectivity of leader–follower dynamical system was investigated in Ref. [3]. MADS can be exactly described by a dynamical network whose nodes denote agents and links represent their changing interactions. Dynamics of first-order differential system have been extensively studied by various approaches [12–21], such as linear matrix inequalities [22] and algebraic graph theorem [23].

Owing to engineering applications, second-order system where agents are governed by both position and velocity states have received considerable interest. Unlike first-order system, Ref. [4] demonstrated underlying network containing directed spanning tree did not guarantee second-order consensus. Some significant consensus conditions were further derived. Ref. [5] proposed constant velocity model. Subsequently, time-varying velocity and nonlinear dynamics were considered in Ref. [6]. Authors in Ref. [7] found both real and imaginary parts of eigenvalues of corresponding Laplacian matrix were closely relative to necessary and sufficient conditions of second-order consensus in MADS.

As we know, impulses exist wildly in nature, some of which are beneficial for collective behaviors while others are not. Ref. [24] proposed an unified criterion which is suit for both synchronizing and desynchronizing impulses. Meanwhile, time-delay is ubiquitous in information transfer. So the study of delayed multi-agent systems with impulsive effects is

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meaningful. Authors in Refs. [20,21,24–29] applied impulsive control to enhance network synchronizability. Second-order impulsive consensus problem was studied in Ref. [30]. It is worthwhile to note dynamically changing topologies in MADS are common due to external perturbations and limited range of communication. Authors in Refs. [19,31,32] investigated consensus problems of first and second-order systems with switching topology.

Besides, Refs. [9,10] proposed an novel collective behaviors of MADS, cluster consensus. This complex system contains many distinct clusters (groups). Cluster consensus describes the phenomenon that agents eventually behave identically if they belong to same cluster, while agents finally have nonidentical states if they belong to different ones. Many collective systems are characterized by such property, such as human, animals. However, it is still a challenging problem to solve cluster consensus in MADS with impulsive effects and switching structures, which motivates this work to propose second-order impulsive consensus criterion.

In this paper, motivated by aforementioned literatures, we discuss the second-order cluster consensus problem of MADS with impulsive effects and coupling delays. How impulses influence on dynamics of MADS with fixed and switching topology are respectively studied. In fixed topology case, beneficial impulsive effects are used to ensure consensus in MADS. In switching case, adaptive method is employed to adjust required control strengths aiming to derive better consensus criteria strategy. Specially, in view of existence of uncertain factors among agents coupling, uncertain interaction topology is established for MADS.

This work is organized as follows. In Section 2, some notations and the model are described. Sufficient conditions for cluster consensus of MADS with fixed and switching topology are stated in Section 3. In Section 4, some numerical examples are given, and conclusion is made in Section 5.

2. Preliminaries

For simplicity, the following notations are used throughout this paper. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is $m\times n$ real matrices. $\mathbb{N}$ denotes non-negative integers. I_n and 0_n are n-order identity matrix and zero matrix, respectively. 1_n is a n-order column vector of all ones. Vector norm is defined as $||x|| = \sqrt{x^T x}, x\in\mathbb{R}^n$. Matrix norm is defined as $||A|| = \sqrt{\rho(A^T A)}, \rho(A)$ is spectral radius of matrix A. For any real symmetric matrix P, notation $P \le 0 (P < 0)$ means P is negative semi-definite (negative definite). Symbol $\otimes$ denotes Kronecker product. $\emptyset$ is emptyset.

Let graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a network, which contains the set of nodes $\mathcal{V} = \{1,\ldots,N\}$; the set of edges $\mathcal{E} = \{e_{ij}\}(i,j\in\mathcal{V})$; and coupling matrices $A = (a_{ij})_{N\times N}$ whose entry a_{ij} denotes connection and weight between agents i and j. If there exists a connection from node j to node i $(i\neq j)$, then $a_{ij}\neq 0$; otherwise, $a_{ij} = 0$. Graph $\mathcal{G}$ denotes the structure of MADS and its weights need not to be positive.

In this paper, we study a MADS composed of N agents, which is split into d disjoint clusters. Denote $\mathcal{C} = \{1,\ldots,d\}$. Sets $\{\mathcal{G}_1,\ldots,\mathcal{G}_d\}$ and $\{\mathcal{V}_1,\ldots,\mathcal{V}_d\}$ are respectively the partition of $\mathcal{G}$ and $\mathcal{V}$ into d subgraph and subsets, i.e., $\bigcup_{r=1}^{d}\mathcal{V}_r = \mathcal{V}$ and $\mathcal{V}_p\bigcap\mathcal{V}_q = \emptyset$ for different p and q. Let symbol $\hat{i}$ denote the cluster where node i belongs to. Thus, $\mathcal{G}_{\hat{i}}$ and $\mathcal{V}_{\hat{i}}$ denote respectively the $\hat{i}$th subgraph and the $\hat{i}$th subset.

We consider a MADS governed by second-order dynamics with impulsive effects

$$\begin{aligned}
&\dot{x}_i(t) = v_i(t), \quad t\neq t_k,\\
&\Delta x_i(t) = x_i(t^+) - x_i(t^-) = w_{ik}(x_i(t) - x_{\hat{i}}(t)), \quad t = t_k,\\
&\dot{v}_i(t) = f_{\hat{i}}(x_i(t), v_i(t), t) + c\sum_{j\in\mathcal{V}} a_{ij}(x_j(t-\tau_{\hat{j}}) + v_j(t-\tau_{\hat{j}})), \quad t\neq t_k,\\
&\Delta v_i(t) = v_i(t^+) - v_i(t^-) = g_{ik}(v_i(t) - v_{\hat{i}}(t)), \quad t = t_k,
\end{aligned} \tag{1}$$

where $(x_i(t) = (x_{i1}),\ldots,x_{in})^T\in\mathbb{R}^n$ and $v_i(t) = (v_{i1},\ldots,v_{in})^T\in\mathbb{R}^n$ are respectively the position and velocity states of the ith agent. $f_{\hat{i}}(x_i(t), v_i(t), t)\in\mathbb{R}^n$ is vector-valued continuous function, c is coupling strength, $\tau_{\hat{i}}$ is time delay, a_{ij} is the (i,j)th entry of delayed coupling matrix A whose entry satisfies $\underline{a}_{ij}\leqslant a_{ij}\leqslant\overline{a}_{ij}$. Matrices $\underline{A} = (\underline{a}_{ij})_{N\times N}$ and $\overline{A} = (\overline{a}_{ij})_{N\times N}$ describe the range of switching structure. Continuous functions $\Phi_i(t) : [-\tau, 0]\to\mathbb{R}^n$ and $\Psi_i(t) : [-\tau, 0]\to\mathbb{R}^n$ $(\tau = \max_{\hat{i}}\tau_{\hat{i}})$ give initial conditions.

The states of agent i will be suddenly changed as result of impulsive distributions at discrete times t_k. $\{t_k|k\in\mathbb{N}\}$ is a time sequence satisfying $0 < \Delta t_k = t_k - t_{k-1} < \infty$ and $\lim_{k\to\infty} t_k\to\infty$. $\{w_{ik}, g_{ik}|i\in\mathcal{V}, k\in\mathbb{N}\}$ are bounded impulsive strengths of agent i at time t_k. We assume that $x_i(t)$ and $v_i(t)$ are left-hand continuous at t_k, i.e., $x_i(t_k) = x_i(t_k^-), v_i(t_k) = v_i(t_k^-)$. Hence, the solutions of system (1) are left continuous at t_k.

Specially, agents belong to same cluster will share same dynamics $f_{\hat{i}}(\cdot)$ and time-delay $\tau_{\hat{i}}$, while in different clusters will not. In each cluster $\mathcal{G}_{\hat{i}}, x_{\hat{i}}(t)\in\mathbb{R}^n$ and $v_{\hat{i}}(t)\in\mathbb{R}^n$ denote respectively position and velocity states of its leader. In this work, we assume that there is a path from leader to each agents in every cluster. The leader in cluster $\mathcal{G}_{\hat{i}}$ is described by

$$\begin{aligned}
&\dot{x}_{\hat{i}}(t) = v_{\hat{i}}(t),\\
&\dot{v}_{\hat{i}}(t) = f_{\hat{i}}(x_{\hat{i}}(t), v_{\hat{i}}(t), t).
\end{aligned} \tag{2}$$

In order to solve the cluster consensus problem in this work, following Assumptions, Definitions and Lemmas are required.