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Local and global stability analysis of a two prey one predator model with help

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ABSTRACT

In this paper we propose and study a three dimensional continuous time dynamical system modelling a three team consists of two preys and one predator with the assumption that during predation the members of both teams of preys help each other and the rate of predation of both teams are different. In this work we establish the local asymptotic stability of various equilibrium points to understand the dynamics of the model system. Different conditions for the coexistence of equilibrium solutions are discussed. Persistence, permanence of the system and global stability of the positive interior equilibrium solution are discussed by constructing suitable Lyapunov functional. At the end, numerical simulations are performed to substantiate our analytical findings.

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1. Introduction

It is not difficult to observe that when the species interacts their dynamics will be affected. Many research have been done on the general problem of food chain. If during interaction of two species the growth rate of one species increases while that of other one decreases then we say that they are in the predator–prey situation. This type of situation arises when one species (predator) feeds on other species (prey). An interesting example of predator–prey model system is the snowy owl, that feeds almost exclusively on the common Arctic rodent called lemming while the lemming uses Arctic tundra plant for it's food supply. The first such fundamental model system representing the interaction between prey and predator species is the Lotka–Volterra model. Lotka–Volterra model was proposed to explain the oscillatory levels of certain fish in the Adriatic sea during the first world war (for details one can refer [8,35]). Now, it has established itself as the simplest and base model system for all the two species competitive model systems [8,10]. A huge amount of mathematical and ecological research have been done dealing with various aspects of predator–prey models. Zhanyuan [4] investigated the permanence behaviour in a Lotka–Volterra system with delays and variable intrinsic growth rates. Yonghui and Maoan [6] gave some new conditions on the existence and stability in a Lotka–Volterra system (one can see [5,7–10,14,26,35]).

There are many factors affecting the dynamics of predator–prey models such as functional response (see, e.g., [18,20,23], competition [9,19], cooperation [33,35], etc. Substantial work have been done to study the stability and other dynamical behaviour of Lotka–Volterra model [11–13]. Many other variants could be found in [14,16–19,23,24,27,28].

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Several investigations have reported complex dynamical behaviour of multi-species food chains of variable lengths (see [10,11,17,20]. As far as models with more than two species are concerned, Freedman and Waltman [29] have discussed persistence behaviour of a general Kolmogorov system (1) of three interacting predator–prey populations:

$$\frac{dx}{dt} = xf_1(x,y,z), \ \frac{dy}{dt} = yf_2(x,y,z), \ \frac{dz}{dt} = zf_3(x,y,z) \tag{1}$$

Freedman and So [32] studied the global stability and persistence behaviour of a simple food chain. In the succession Farkas and Freedman [15] found stability conditions for a two predator and one prey system. In the subsequent paper Dubey and Upadhyay [34] studied persistence and extinction of one-prey and two-predators model system. Gakkhar and Singh [23] discussed the dynamics of a two preys and a harvesting predator. Some other works related to the dynamical behaviour of models modelling more than two species can be found in [30,33,35]. For the analytical study of persistence and permanence, one can also see [1–3,31,36].

Many times animals form a team, a good introduction to teaming approach can be found in [21,22,25]. Naturally all the species want to survive in the wild. Some do that by living alone. Others live in flocks, herds, hives, packs, or schools. For some animals, the best way to stay alive is to live with or near other animals. The needs of the team are best met when the needs of individual member are fulfilled. Forming team and cooperating others is one of the basic tool through which a member of the team always get good results and fulfils their needs easily. The two main advantages due to teaming are:

(i) Improvement in foraging since looking for food in a team is more efficient than doing it alone.
(ii) Living together gives a higher probability that the predator will attack another member of the team.

The proposed model is motivated by two prey one predator model given in [33,35]. We consider the following two important and interesting factors in the proposed model system (5):

(a) Different predation rates α_1 and α_2 for both the prey teams.
(b) The prey teams may interact in the absence of predator with different rates of competition γ_1 and γ_2.

In this paper we propose and analyze different dynamical behaviour mathematically and numerically of a competitive two-prey (team) one predator (team) model system with help. We determine conditions and establish analytical results supported by numerical simulations for persistence, permanence, boundedness, local stability and the global stability of the coexisting equilibrium solutions.

The paper is structured as follows. Some preliminary results used in this study are given in Section 2. In Section 3, we describe and discuss the basic model system and permanence, persistence and related dynamical behaviour of the model system (5). Numerical simulations to validate analytical results are prescribed in Section 4. In the last section, the results of analytical findings are discussed in the context of biological realization.

2. Preliminaries

In this section we mention some definitions and preliminaries which are used in further discussion.

Definition 2.1 (*Generalized Eigen Vector*). A generalized eigen vector of a matrix A is a non-zero vector v which is associated with eigen value λ, having algebraic multiplicity k and satisfies:

$$(A - \lambda I)^k v = 0 \tag{2}$$

Definition 2.2 (*Stable, Unstable and Center subspaces*). Subspaces which are spanned by the generalized eigen vectors corresponding to the eigen value λ with $Re(\lambda) < 0$, $Re(\lambda) > 0$, $Re(\lambda) = 0$, are called stable, unstable and center subspaces respectively.

Definition 2.3 (*Permanence*). System (5) is said to be permanent if $\exists$ positive constants m, m^* and M such that $m \leqslant x(t), \ m^* \leqslant y(t), \ z(t) \leqslant M, \ \forall t \geqslant 0$, where $(x(t), y(t), z(t))$ denote any solution of (5).

Definition 2.4 (*Persistence*). System (5) is said to be persistent if every solution (x, y, z) satisfies two conditions:

(i) $x(t) \geqslant 0, \ y(t) \geqslant 0, \ z(t) \geqslant 0, \forall t \geqslant 0$.
(ii) $\liminf_{t\to\infty} x(t) > 0, \ \liminf_{t\to\infty} y(t) > 0$ and $\liminf_{t\to\infty} z(t) > 0$.

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