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Global resonance optimization analysis of nonlinear mechanical systems: Application to the uncertainty quantification problems in rotor dynamics

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ABSTRACT

An efficient method to obtain the worst quasi-periodic vibration response of nonlinear dynamical systems with uncertainties is presented. Based on the multi-dimensional harmonic balance method, a constrained, nonlinear optimization problem with the nonlinear equality constraints is derived. The MultiStart optimization algorithm is then used to optimize the vibration response within the specified range of physical parameters. In order to illustrate the efficiency and ability of the proposed method, several numerical examples are illustrated. The proposed method is then applied to a rotor system with multiple frequency excitations (unbalance and support) under several physical parameters uncertainties. Numerical examples show that the proposed approach is valid and effective for analyzing strongly nonlinear vibration problems with different types of nonlinearities in the presence of uncertainties.

1. Introduction

The method of determining periodic solutions of nonlinear systems is one of the most important fields in nonlinear dynamics researches. Many approaches have been developed to approximate periodic solutions of nonlinear systems. The harmonic balance method (HBM) is a well-known method to obtain approximate periodic solutions of nonlinear differential equations by using a truncated Fourier series. Many variants on the HBM have emerged. For example, Cheung and Lau [1] proposed the incremental harmonic balance method. Cameron and Griffin [2] pioneered the development of the alternating frequency/time domain(AFT) method. Based on augmented Lagrangian to deal with nonsmooth contact-friction laws, Nacivet et al. [3] proposed the dynamic Lagrangian frequency–time (DLFT) method. This method has been successfully used to quantify the efficiency of friction ring dampers [4]. Hall et al. [5] developed an improved version of the HBM which is referred to as the high dimensional harmonic balance (HDHB) approach in this study. The HDHB method was successfully improved to investigate the aeroelastic motions of an airfoil in Ref. [6]. In the HDBD method, aliasing [7] can occur if nonlinearity introduces a higher harmonic order. LaBryer described this phenomenom in Ref. [8] and proposed filters to decrease aliasing. Cochelinetal et al. [9] presented another strategy for applying the classical HBM with a large number of harmonics. The basic idea consists in recasting the original system into a new system where nonlinearities are at most quadratic polynomials by introducing as many new variables as needed. Recently, a constrained harmonic balance method [10] which computes solutions for periodic autonomous systems has been proposed. In the constrained harmonic balance method, a

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square system of nonlinear equations is generated by setting an unknown variable to a given value so that the size of the nonlinear algebraic equations is as many as unknowns. In order to estimate the number of selected harmonics for a given level of accuracy and reduce the size of the set of nonlinear algebraic equations, an adaptive harmonic balance method was developed in Ref. [11]. A global analysis method mixing a harmonic balance method and a homotopy technique is presented in Ref. [12] to find all the periodic solutions of a nonlinear system.

By making use of the harmonic balance method, a set of nonlinear algebraic equations is formed and is therefore well-determined. A nonlinear solver is then used to find zeros of such nonlinear equations. However, in order to perform parametric studies the nonlinear solver must be applied recursively. In addition, if the number of unknown variables is higher than the number of the nonlinear harmonic balance equations (such as structural parameters uncertainties), the nonlinear harmonic balance equations are the so called under-defined system which cannot be solved using the nonlinear root finding algorithm. Furthermore, the location of the nonlinear resonance extremum is a difficulty problem (see Page 1144 in Ref. [13]) Therefore, there is a need to locate the nonlinear resonance extremum and the nonlinear resonant frequency.

As mentioned previously, the harmonic balance method with a single fundamental frequency is limited to find periodic solutions and is unfit to find quasi-periodic solutions. The single-frequency harmonic balance method is therefore extended to the case where the solution is not periodic in time but is quasi-periodic. The first formal presentation of the harmonic balance method to obtain the quasi-perioidic solutions of nonlinear systems is usually credited to Chua and Ushida [14]. Following the pioneer work of Chua and Ushida [14] in electronics, a similar approach named the incremental harmonic balance method with multiple time scales was reported by Lau et al. [15] and it was further developed in Ref. [16] to handle general multi-degree of freedom externally excited and autonomous dynamical systems with cubic nonlinearities. By utilizing the AFT method in conjunction with multi-frequency Fourier transforms, the multi-dimensional harmonic balance method (MHBM) was proposed in Ref. [17][18] to study the internal resonant vibrations of a nonlinear Jeffcott rotor with contact terms. In Ref. [19] the multi-dimensional harmonic balance method with arc-length continuation was extended to the nonlinear vibration analysis of a modified Jeffcott rotor with piece-wise radial stiffness. Based on the approximation of the frequency basis by a mono-dimensional one, a new methodology called the adjusted harmonic balance method has been introduced in Ref. [20].

Most structural and mechanical systems are subject to variability and uncertainty in real life. Thus, the performance characteristics of such systems are also subject to uncertainties. In structural dynamics, taking into account uncertainty is important for various reasons: to increase the robustness of design, to ensure the compliance of vibration levels to standards, to assess worst case behavior and so on. In many instances, simulating solutions using a deterministic model may lead to inaccurate computational results. Therefore, any realistic analysis of nonlinear systems must take the uncertainties into account. The uncertainty analysis of nonlinear systems has received considerable attention. The Monte Carlo simulation (MCS) is a standard technique which has been widely used in engineering community for stochastic simulations. However, MCS requires a tremendous computational cost.

There is a growing interest to develop efficient computational algorithms for uncertainty investigations in nonlinear systems. Different computational methodologies have been developed to quantify the uncertain response of nonlinear systems subject to parametric variability. The uncertainty in nonlinear systems can be modeled using probabilistic, interval or other methods. When the uncertain parameters are described as random variables with known probability distributions, the probabilistic methods can be used. In recent years, polynomial chaos expansion, which helps to describe random functions with convergent polynomial functions series in some independent random variables with joint density functions, has been applied to many engineering problems taking into account the effects due to uncertainty. In Refs. [21,22], the polynomial chaos expansion with the harmonic balance method is presented and applied to predict the dynamic behaviors in uncertain rotor systems with faults. By incorporating the polynomial chaos expansion with the multi-dimensional harmonic balance method, Didier [23]developed the Stochastic multi-dimensional harmonic balance method. Unfortunately, the probabilistic approaches require a wealth of data, often unavailable, to define the probability density function of the uncertain structural parameters. Furthermore, Millman et al. [24] found that the PCE fail to predict limit cycle oscillations.

In many real situations, the maximum possible ranges of variations expressed in terms of percentage of the corresponding nominal of the parameters are only known and can be modeled as uncertain but bounded type parameters. In such cases, the interval analysis method [25] is a viable alternative. However, the interval analysis often leads to an overestimation of the interval width [26]. Hence, reducing the overestimation in the interval method is a crucial issue to a successful interval analysis. To overcome these limitations, the objective of the present work is to develop a systematic methodology to determine the quasi-periodic solutions of nonlinear systems in the presence of uncertainty.

Remainder of this paper is organized as follows: the general formulation of the proposed method for determining the quasi-periodic solutions is presented in Section 2. Examinations of the proposed method using a rotor system are then conducted in Section 3. Finally, concluding remarks are presented and discussed in Section 4.

2. The proposed method

A method named constrained optimization multi-dimensional harmonic balance method (COMHBM) which combines the multi-dimensional harmonic balance method and the MultiStart optimization algorithm is proposed in this section. The proposed method is formulated as a constrained, nonlinear optimization problem. This paper is devoted to study the