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Transient times, resonances and drifts of attractors in dissipative rotational dynamics

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ABSTRACT

In a dissipative system the time to reach an attractor is often influenced by the peculiarities of the model and in particular by the strength of the dissipation. As a dissipative model we consider the spin–orbit problem providing the dynamics of a triaxial satellite orbiting around a central planet and affected by tidal torques. The model is ruled by the oblateness parameter of the satellite, the orbital eccentricity, the dissipative parameter and the drift term. We devise a method which provides a reliable indication on the transient time which is needed to reach an attractor in the spin–orbit model; the method is based on an analytical result, precisely a suitable normal form construction. This method provides also information about the frequency of motion. A variant of such normal form used to parameterize invariant attractors provides a specific formula for the drift parameter, which in turn yields a constraint – which might be of interest in astronomical problems – between the oblateness of the satellite and its orbital eccentricity.

1. Introduction

We consider a nearly-integrable dissipative system describing the motion of a non-rigid satellite under the gravitational influence of a planet. The motion of the satellite is assumed to be Keplerian; the spin–axis is perpendicular to the orbit plane and it coincides with the axis whose moment of inertia is maximum. The non-rigidity of the satellite induces a tidal torque provoking a dissipation of the mechanical energy. The dissipation depends upon a dissipative parameter and a drift term. If the dissipation were absent, the system becomes nearly-integrable with the perturbing parameter representing the equatorial oblateness of the satellite. The overall model depends also on the orbital eccentricity of the Keplerian ellipse. This problem is often known as the *dissipative spin–orbit model* and it has been extensively studied in the literature (see, e.g., [5,7,22]).

The spin–orbit model exhibits different kinds of attractors, e.g. periodic, quasi-periodic and strange attractors (compare with [1,2,9,15]). As it often happens in dissipative system, the dynamics evolves in such a way that the attractor is reached after an initial transient regime of motion. The prediction of the transient time to reach the attractor is often quite difficult (see, e.g., [18,19]), but it is obviously of pivotal importance to test the reliability of the result (think, e.g., to the problem of deciding about the convergence of the Lyapunov exponents). The first goal of this paper is to give a recipe which allows to decide the length of the transient time, namely the time needed to go over the transient regime and to settle the system into its typical behavior. Our study is based on the construction of a suitable normal form for dissipative vector fields (see [8], compare also with [13,16,20,24]) that generalizes Hamiltonian normal forms that are usually implemented around elliptic

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equilibria (see [14]). We compute the frequency in the normalized variables and use it – as well as its back-transformation to the original variables – for a comparison with a numerical integration of the equations of motion. Several experiments are performed as the strength of the dissipation is varied. It should be kept in mind that in dissipative systems one has to tune the drift parameter in order to get specific attractors, since it does not suffice to modify the initial conditions like in the conservative case ([3,6]). A different formulation of the normal form, precisely a suitable parametric representation of invariant attractors, allows to obtain an explicit form for the drift on the attractor. Taking advantage of the physical definition of the drift term, precisely as a function of the eccentricity ([23], see also [11]), one can derive interesting conclusions on a link between the oblateness parameter and the eccentricity associated to a given invariant attractor. We believe that this constraint might be useful in concrete astronomical applications.

This paper is organized as follows. In Section 2 we present the equations of motion of the spin–orbit problem in the conservative and dissipative cases. The construction of the normal form is developed in Section 3, while the parametric representation of invariant attractors is provided in Section 4. The investigation of the transient time and the analysis of the drift term are performed in Section 5. Some conclusions are drawn in Section 6.

2. The spin–orbit problem with tidal torque

In this Section we describe the spin–orbit model, providing the equation of motion in the conservative case (Section 2.1) and under the effect of a tidal torque, due to the internal non-rigidity of the satellite (Section 2.2).

2.1. The conservative spin–orbit problem

The spin–orbit model describes the dynamics of a rigid body with mass m, say $\mathcal{S}$, that we assume to have a triaxial structure with principal moments of inertia $I_1 \leqslant I_2 \leqslant I_3$. The satellite $\mathcal{S}$ moves under the gravitational effect of a perturbing body $\mathcal{P}$ with mass M. Moreover, we make the following assumptions:

(i) the body $\mathcal{S}$ orbits on a Keplerian ellipse around $\mathcal{P}$; we denote by a and e the corresponding semimajor axis and eccentricity;
(ii) the rotation axis of $\mathcal{S}$ is assumed to coincide with the direction of the largest principal axis of inertia;
(iii) the spin–axis is assumed to be aligned with the orbit normal;
(iv) all other perturbations, including dissipative effects, are neglected.

In order to simplify the notation, we normalize the units of measure; precisely, the mean motion $\frac{\mathcal{G}M}{a^3}$ (where $\mathcal{G}$ is the gravitational constant) is normalized to one. An important role is played by the following quantity, which is named the *equatorial ellipticity:*

$$\varepsilon \equiv \frac{3}{2}\frac{I_2 - I_1}{I_3}.$$

To describe the rotation of $\mathcal{S}$ with respect to $\mathcal{P}$, we introduce the angle x spanned by the largest physical axis (that we assume to lie in the orbital plane) with the perihelion line (see Fig. 1).

The Hamiltonian function describing the spin–orbit model under the assumptions (i)–(iv) is (see [5])

$$\mathcal{H}(y, x, t) = \frac{y^2}{2} - \frac{\varepsilon}{2}\left(\frac{a}{r}\right)^3 \cos(2x - 2f), \tag{1}$$

where y is the momentum conjugated to x, r is the orbital radius and f is the true anomaly.

Hamilton's equations associated to (1) are given by

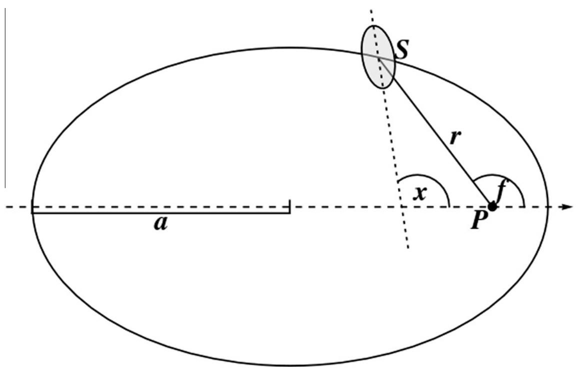

Fig. 1. The geometry of the spin–orbit problem: orbital radius r, semi-major axis a, true anomaly f, rotation angle x.