Commun Nonlinear Sci Numer Simulat 15 (2010) 1813–1830

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction

Dulal Pal [a,*], Babulal Talukdar [b]

[a] Department of Mathematics, Visva-Bharati University, Santiniketan, West Bengal 731 235, India
[b] Department of Mathematics, Gobindapur High School, Kalabagh, Murshidabad, West Bengal 742 225, India

ARTICLE INFO

Article history:
Received 13 March 2009
Received in revised form 13 July 2009
Accepted 14 July 2009
Available online 17 July 2009

Keywords:
Thermal radiation
Heat transfer
Porous medium
Boundary layer flow
Slip flow
First-order chemical reaction

ABSTRACT

An analytical study for the problem of unsteady mixed convection with thermal radiation and first-order chemical reaction on magnetohydrodynamics boundary layer flow of viscous, electrically conducting fluid past a vertical permeable plate has been presented. Slip boundary condition is applied at the porous interface. The classical model is used for studying the effect of radiation for optically thin media. The non-linear coupled partial differential equations are solved by perturbation technique. The results obtained show that the velocity, temperature and concentration fields are appreciably influenced by the presence of chemical reaction, thermal stratification and magnetic field. It is observed that the effect of thermal radiation and magnetic field decreases the velocity, temperature and concentration profiles in the boundary layer. Also, the effects of the various parameters on the skin-friction coefficient and the rate of heat transfer at the surface are discussed.

1. Introduction

Mixed convection flows with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction arise in many transport processes both naturally and in many branches of science and engineering applications. They play an important role in many industries viz. in the chemical industry, power and cooling industry for drying, chemical vapour deposition on surfaces, cooling of nuclear reactors and magnetohydrodynamic (MHD) power generators. Such processes occur when the effects of buoyancy forces in forced convection or the effects of forced flow in free convection become significant. Many transport process exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of diffusion of chemical species. A few fields of interest in which combined heat and mass transfer plays an important role in design of chemical process in equipment, formation and dispersion of fog, distribution of temperature and moisture over agriculture fields. A comprehensive description of the theoretical work for both laminar and turbulent mixed convection boundary layer flows has been given in a review paper by Chen and Armaly [1] and in the book by Pop and Ingham [2]. Magnetohydrodynamics plays an important role in agriculture, engineering and petroleum industries. The problem of mixed convection under the influence of magnetic field has attracted numerous researchers in view of its applications in geophysics and astrophysics. Soundalgekar et al. [3] investigated the problem of free convection effects on Stokes problem for a vertical plate with transverse applied magnetic field whereas Elbasheshy

* Corresponding author.
E-mail address: dulalp123@rediffmail.com (D. Pal).

1007-5704/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2009.07.011

[4] studied MHD heat and mass transfer problem along a vertical plate under the combined buoyancy effects of thermal and species diffusion. Abel et al. [5] analyzed the effects of thermal buoyancy and variable thermal conductivity on the MHD flow and heat transfer in a power-law fluid past a vertical stretching sheet in the presence of a non-uniform heat source. Another related paper by Abel et al. [6] is on flow and heat transfer in a power-fluid over a stretching sheet with the variable thermal conductivity and non-uniform heat source.

In the above mentioned studies the radiation effect is ignored. When technological processes take place at high temperatures thermal radiation heat transfer become very important and its effects cannot be neglected (Siegel and Howell [7]; Modest [8]). Recent developments in hypersonic flights, missile reentry rocket combustion chambers, gas cooled nuclear reactors and power plants for inter planetary flight, have focused attention of researchers on thermal radiation as a mode of energy transfer, and emphasize the need for inclusion of radiative transfer in these process. The interaction of radiation with mixed convection flows past a vertical plate was investigated by Hossain and Takhar [9]. Aboeldahab [10] studied the radiation effect in heat transfer in an electrically conducting fluid at stretching surface. Heat and mass transfer effects on moving plate in the presence of thermal radiation have been studied by Muthucumarswamy and Kumar Senthil [11] using Laplace technique. Abo-Eldahab and Elgendy [12] presented radiation effect on convective heat transfer in an electrically conducting fluid at a stretching surface with variable viscosity and uniform free stream. Seddeek [13] examined the effect of radiation and variable viscosity on unsteady forced convection flows in the presence of an align magnetic field. Abdus Satter and Hamid Kalim [14] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. Cortell [15] studied the effects of viscous dissipation and radiation on the thermal boundary layer over a non-linearly stretching sheet. Abel and Mahesha [16] investigated heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Recently, Kim [17] discussed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction.

Convection in porous media has gained significant attention in recent years because of its importance in engineering applications such as geothermal systems, solid matrix heat exchangers, thermal insulations, oil extraction and store of nuclear waste materials. Convection in porous media can also be applied to underground coal gasification, ground water hydrology, iron blast furnaces, wall cooled catalytic reactors, solar power collectors, energy efficient drying processes, cooling of nuclear fuel in shipping flasks, cooling of electronic equipments and natural convection in earth's crust. Reviews of the applications related to convective flows in porous media can be found in Nield and Bejan [18]. The fundamental problem of flow through and past porous media has been studied extensively over the years both theoretically and experimentally [19,20]. The inadequacy of the no-slip condition is quite evident in polymer melts which often exhibit microscopic wall slip. The boundary conditions to be satisfied at the interface between a porous medium and fluid layer are the matching of velocity and stresses. Beavers and Joseph [21] were the first to investigate the fluid flow at the interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary conditions at the porous interface. A theoretical justification, using a statistical approach, of the boundary conditions of Beavers–Joseph (BJ) was given by Saffman [22]. Most available studies on this problem are concerned with the case of horizontal fluid layer bounded by rigid impervious walls. However, in many applications, the convective flow is in interaction with more complex boundary conditions such as by porous layers for instance. The use of Brinkman equation for the porous layer with the continuity of velocity and shear at the interface condition is to made. The study in flow past a porous medium is usually based on Darcy's equation with the slip conditions at the interface as proposed by Beavers and Joseph [21]. Nield [23,24] applied the Beavers and Joseph (BJ) boundary condition to study the thermal stability of superposed porous and fluid layers for various boundary conditions at the upper and lower of the system. The Beavers–Joseph model has also been used to investigate natural convection within systems of superposed porous and fluid layers. Poulikakos et al. [25] reported a numerical analysis of high Rayleigh convection for the case of a fluid layer on top of a porous layer. Rudraiah et al. [26] studied the effect of BJ slip velocity and transverse magnetic field on an electrically conducting viscous fluid in a horizontal channel bounded on both sides by porous substrates of finite thickness which is analogous to the problem of forced convection where the momentum equation is independent of concentration distribution and the diffusion equation is coupled with the velocity distribution using BJ slip condition at the porous interface.

The combined effects of heat and mass transfer with chemical reaction are of great importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering, and hence received a considerable amount of attention in recent years. The study of chemical reaction with heat transfer in porous medium has important engineering applications e.g. tubular reactors, oxidation of solid materials and synthesis of ceramic materials. There are two types of reactions such as (i) homogeneous reaction and (ii) heterogeneous reaction. A homogeneous reaction occurs uniformly throughout the given phase, whereas heterogeneous reaction takes place in a restricted region or within the boundary of a phase. The effects of a chemical reaction depends on whether the reaction is heterogeneous or homogeneous. A chemical reaction is said to be first-order, if the rate of reaction is directly proportional to concentration itself. In many industrial process involving flow and mass transfer over a moving surface, the diffusing species can be generated or absorbed due to some kind of chemical reaction with the ambient fluid which can greatly affect the flow and hence the properties and quality of the final product. These processes take place in numerous industrial applications, such as the polymer production and the manufacturing of ceramics or glassware. Thus we are particularly interested in cases in which diffusion of the species and chemical reaction occur at roughly the same speed in analyzing the mass transfer phenomenon. Das et al. [27] have studied the effect of homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate