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Synchronization of hyperchaotic systems via linear control ☆

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ABSTRACT

In this paper, synchronization of hyperchaotic system is discussed. Based on the stability theory in the cascade system, a simple linear feedback law is presented to realize synchronization of hyperchaotic systems. Simulation results are given to illustrate the effectiveness of the proposed method.

1. Introduction

Chaotic phenomenon has received increasing attention in the past several decades. Compared with the ordinary chaotic systems, hyperchaotic systems hold more than one positive Lyapunov exponents. Hence, hyperchaotic systems possess more complicated attractors. Since hyperchaotic systems have the characteristics of high capacity, high security and high efficiency, they have been researched more and more in the fields of secure communication, information processing, biological engineering, chemical processing and other fields [1–9].

Many methods have been proposed to realize chaos synchronization in low dimensional attractors with one positive Lyapunov exponent. However, chaotic system with higher dimensional attractor have much wider application. The presence of more than one Lyapunov exponent clearly improves security of the communication scheme by generating more complex dynamics. So, hyperchaotic systems are being given more and more interest [10–13]. Several methods such as nonlinear control method [11], linear control method [12], adaptive control method [13,14], neural networks method [15] have been provided for hyperchaotic synchronization.

Studies on the stabilization of cascade systems have attracted many researchers' attention in nonlinear control field [16,17]. Among theses advances are several constructive design methods such as backstepping and forwarding, which are based on recursive applications of cascade designs. From some aspects, the hyperchaotic system can be seen as cascade system. In this paper, linear feedback law is designed based on the stability theory in cascade system. Taking some error states as virtual control inputs, input-to-state stability (ISS) theory [18,19] is technically implemented to attain global asymptotical stability of the overall error system. Linear controller is designed step by step and the feedback control obtained in this way is simple. It can be easily implemented in the practical process.

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2. Preliminary results and lemmas

Before giving the main results, let us introduce some necessary lemmas.

Definition 1. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class K if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class K_∞ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

Definition 2. A continuous function $\beta : [0, a] \times [0, \infty) \to [0, \infty)$ is said to belong to class KL if, for each fixed s, the mapping $\beta(r, s)$ belongs to class K with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Lemma 1 [20]. *Let $x = 0$ be an equilibrium point for $\dot{x} = f(t, x)$ and $D \subset R^n$ be domain containing $x = 0$. Let $V : [0, \infty) \times D \to R$ be a continuously differentiable function such that*

$$\begin{aligned} &\alpha_1 \|x\|^\beta \leqslant V(t, x) \leqslant \alpha_2 \|x\|^\beta, \\ &\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\dot{x} \leqslant -\alpha_3 \|x\|^\beta, \end{aligned} \tag{1}$$

$\forall t \geqslant 0$ and $\forall x \in D$, where $\alpha_i (i = 1, 2, 3)$ and β are positive constants. Then, $x = 0$ is exponentially stable. If the assumptions hold globally, then $x = 0$ is globally exponentially stable.

Lemma 2 [20]. *Consider the system*

$$\dot{x} = f(t, x, u). \tag{2}$$

Suppose $f : [0, \infty) \times R^n \times R^m \to R^n$ is continuously differentiable and globally Lipschitz in (x, u), uniformly in t. If the unforced system $\dot{x} = f(t, x, 0)$ has a globally exponentially stable equilibrium point at the origin $x = 0$, then the system (2) is input-to-state stable.

Lemma 3 [20]. *Consider the following system*

$$\dot{x}_1 = f_1(t, x_1, x_2), \tag{3}$$

$$\dot{x}_2 = f_2(t, x_2), \tag{4}$$

where $f_1 : [0, \infty) \times R^{n_1} \times R^{n_2} \to R^{n_1}$ and $f_2 : [0, \infty) \times R^{n_2} \to R^{n_2}$ are piecewise continuous in t and Locally Lipschitz in $x = [x_1, x_2]^T$. If the system (3), with x_2 as input, is input-to-state stable and the origin of (4) is globally uniformly asymptotically stable, then the origin of the cascade system (3) and (4) is globally uniformly asymptotically stable.

Proof. By the definition of asymptotical stability and the definition of ISS, the condition of Lemma 3 implies that there exist KL-class functions β_1 and β_2, and a K-class function γ_1 such that for any $t \geqslant s \geqslant t_0$,

$$\|x_1(t)\| \leqslant \beta_1(\|x_1(s)\|, t - s) + \gamma_1\left(\sup_{s \leqslant \tau \leqslant t} \|x_2(\tau)\|\right), \tag{5}$$

$$\|x_2(t)\| \leqslant \beta_2(\|x_2(s)\|, t - s), \tag{6}$$

Let $s = \frac{t+t_0}{2}$. It follows $t \geqslant s = \frac{t+t_0}{2}$. Because $t \geqslant s \geqslant t_0$, (5) becomes

$$\|x_1(t)\| \leqslant \beta_1\left(\left\|x_1\left(\frac{t+t_0}{2}\right)\right\|, \frac{t-t_0}{2}\right) + \gamma_1\left(\sup_{\frac{t+t_0}{2} \leqslant \tau \leqslant t} \|x_2(\tau)\|\right). \tag{7}$$

Using (5) again, but now t is replaced by $\frac{t+t_0}{2}$ and $s = t_0$, we have

$$\left\|x_1\left(\frac{t+t_0}{2}\right)\right\| \leqslant \beta_1\left(\|x_1(t_0)\|, \frac{t-t_0}{2}\right) + \gamma_1\left(\sup_{t_0 \leqslant \tau \leqslant \frac{t+t_0}{2}} \|x_2(\tau)\|\right). \tag{8}$$

By (6),

$$\|x_2(t)\| \leqslant \beta_2(\|x_2(t_0)\|, t) \leqslant \beta_2(\|x_2(t_0)\|, 0). \tag{9}$$

Note that the right side is a constant, hence

$$\sup_{t_0 \leqslant \tau \leqslant \frac{t+t_0}{2}} \|x_2(\tau)\| \leqslant \beta_2(\|x_2(t_0)\|, 0), \tag{10}$$

By (10), (8) leads to