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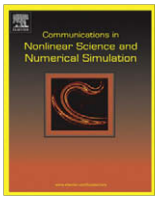

Short communication

Viscous flow over a shrinking sheet with an arbitrary surface velocity

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A B S T R A C T

In this paper, an analytical solution in a closed form for the boundary layer flow over a shrinking sheet is presented when arbitrary velocity distributions are applied on the shrinking sheet. The solutions with seven typical velocity profiles are derived based on a general closed form expression. Such flow is usually not self-similar and the solution can only be implemented when the mass transfer at the wall is prescribed and determined by the moving velocity of the wall. The characteristics of the flows with the typical velocity distributions are discussed and compared with previous similarity solutions. The flow is observed to have quite different behavior from that of the self-similar flow reported in the literature and the results demonstrate distinctive momentum and energy transport characteristics. Some plots of the stream functions are also illustrated to show the difference in flow field between the shrinking sheet and the stretching sheet. An integral approach to solve boundary layer flow over a shrinking or stretching sheet with uncoupled arbitrary surface velocity and wall mass transfer velocity is outlined and the effectiveness of this approach is discussed.

1. Introduction

The flow over a continuously stretching surface can be found in many industrial processes [1–3]. This type of flow was firstly investigated by Sakiadis over a continuously stretching surface with a constant speed [4,5]. Numerous studies have been conducted afterward to explore the flow characteristics for various applications [6–18]. Most of the solutions ,except the one by Crane [7], obtained in the previous works, are based on the boundary layer assumption and are not exact solutions of the Navier-Stokes (NS) equations [19]. Furthermore, various forms of solutions have been proposed and multiple solution branches for both impermeable and permeable stretching sheets were also discovered for some cases [17,18]. Kumaran and Ramamaiah presented a close-form solution to this flow when the stretching velocity of the wall is parabolic [20]. Recently, the solution to the flow induced by continuous stretch of the surfaces, originally presented by Crane [7], has been generalized for the walls with an arbitrary shrinking velocity [21]. This work is an extension to the study by Kumaran and Ramanaiah [20] and provides solution to an arbitrarily high degree of polynomial stretching. Miklavcic and Wang [22] initiated the study on the flow over a shrinking sheet and presented exact solutions of the NS equations. The shrinking sheet problem has been receiving much more attention in the literature since then and the work on the flow configurations have been extended to power-law shrinking velocity and to various fluids [23–29]. All the previous work has shown that the mass transfer from a permeable wall is required in order to maintain the flow over a shrinking sheet. However, the analytical solution to the flow over a shrinking sheet with an arbitrary surface velocity has not reported. The objective of this paper is to extend the shrinking sheet problem to any arbitrary shrinking velocity. The solution will be presented in a closed form and different velocity

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distributions at the wall, including constant, linear, quadratic, power-law, exponential, periodic and bilinear velocity, will be demonstrated as examples of this solution to show the flow characteristics. The heat transfer aspect of the problem with boundary layer assumption will be discussed as well.

2. Mathematical formulation and solutions

2.1. Flow configuration

Consider a steady, two-dimensional laminar flow over a continuously shrinking sheet in a quiescent fluid. The sheet shrinking velocity is $u_w = -u_w(x)$ with $u_w(x)$ being positive for all values of x and the mass transfer velocity at the wall is $v_w = v_w(x)$, which will be determined later. The x-axis runs along the shrinking surface in a direction opposite to the sheet motion and the y-axis is perpendicular to the shrinking surface. The governing momentum and energy equations based on the boundary layer assumption read [21]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the boundary conditions (BCs)

$$u(x,0) = -u_w(x), \quad v(x,0) = v_w(x), \quad u(x,\infty) = 0, \tag{4a–c}$$

and

$$T(x,0) = T_w(x), \quad T(x,\infty) = T_\infty \tag{5a–b}$$

where u and v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, p is the fluid pressure, ρ is the fluid density, T is the fluid temperature, α is the thermal diffusivity of the fluid, and subscript w denotes the conditions at the wall. In the following sections, a closed form solution will be given first and some examples of various velocity distributions will be presented.

2.2. Closed form solutions

The governing Eqs. (1)–(3) can be transformed into dimensionless forms by using a characteristic velocity and a length scale. By defining $U(X,Y) = u(x,y)/U_0,\ V(X,Y) = v(x,y)/U_0,\ P(X,Y) = p(x,y)/\left(\rho U_0^2\right),\ X = x/L, Y = y/L$ with $L = \nu/U_0$, the dimensionless momentum equations become

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{6}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} \tag{7}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial Y^2} \tag{8}$$

with the boundary conditions (BCs)

$$U(X,0) = -U_w(X), \quad V(X,0) = V_w(X), \quad U(X,\infty) = 0. \tag{9a–c}$$

and

$$T(X,0) = T_w(X), \quad T(X,\infty) = T_\infty \tag{10a–b}$$

where $Pr = \nu/\alpha$ is the Prandtl number and U_0 is a constant characteristic velocity.

The momentum and the continuity equations admit the following solutions:

$$U(X,Y) = -U_w(X)e^{-\beta Y} \tag{11}$$

and

$$V(X,Y) = -\beta - \frac{1}{\beta}\frac{dU_w(X)}{dX}e^{-\beta Y}. \tag{12}$$

The energy equation has a particular solution as

$$T(X,Y) = T_\infty + T_r[U_w(X)]^{Pr}e^{-\beta Pr Y} \tag{13}$$