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Short communication

Abundant exact solutions for the higher order non-linear Schrödinger equation with cubic–quintic non-Kerr terms

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ABSTRACT

In this paper, the higher order NLS equation with cubic–quintic non-linear terms arising in non-Kerr media is studied, new abundant solitary solutions of this equation are obtained using generalized auxiliary equation method.

1. Introduction

As we know that non-linear Schrödinger (NLS) equation

$$iu_t + \gamma u_{xx} + \rho|u|^2u = 0.$$

is a fundamental model for the non-linear propagation of light pulses in optical fibers and a completely integrable system by the inverse scattering transform [1]. In this paper, we consider a generalization of the NLS equation researched by Radhakrishnan, Kundu and Laskshmanan (RKL model for short), which presented the higher order NLS equation with cubic–quintic non-linear terms arising in non-Kerr media [2]. However, the RKL model and the general higher order NLS models proposed are not completely integrable and cannot be exactly solved by the inverse scattering transform method [3]. The RKL model [4–6]

$$iu_t + u_{xx} + \tau|u|^2u + \gamma|u|^4u + i\gamma_1 u_{xxx} + i\gamma_2\left(|u|^2u\right)_x + i\gamma_3\left(|u|^4u\right)_x = 0. \quad (1)$$

where $\gamma,\ \gamma_1,\ \gamma_2,\ \gamma_3$ and τ are real constants, $u(x,t)$ is a complex function,the coefficient γ_2 and γ_3 represent the self-steepening term for short pulses, γ and τ represent the coefficient of cubic–quintic terms, describing the effects of quintic non-linearity on the ultrashort optical soliton pulse propagation in non-Kerr media [8,11,12]. Physically, power law non-linearity arises in various materials, including semiconductors. Moreover, this law of non-linearity arises in non-linear plasmas that solves the problem of small K-condensation in weak turbulence theory [11,12]. The coefficient of third order dispersion term

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is given by γ_1. For the RKL model, Zhang and Wang [7] obtained the various exact solutions of two special type RKL models by using of the elliptic-like subsidiary ordinary differential equation and the sine–cosine method, Triki and Taha [8] fund the exact analytic solitary wave solutions with the Jacobi elliptic function method. Hong [3] has investigated general analytical solitary wave solutions and found explicit dark and bright soliton solutions by adopting the complex amplitude ansatz method introduced by Li et al. [9]. In this paper, we obtain directly the abundant exact solitary wave solutions using a new envelope transform and new results of Lienard equation [10].

2. Abundant solutions

We assume the solution $u(x,t)$ of (1) has the following form:

$$u(x,t) = \phi(\xi)e^{i\eta}, \quad \xi = \alpha t - wx, \quad \eta = kt - cx, \tag{2}$$

where $\phi(\xi)$ is a real function and $\alpha,\ w,\ k,\ c$ are real constants to be determined later. Substituting (2) into (1) and separating the real and imaginary part, we have

$$-(k + c^2 + \gamma_1 c^3)\phi(\xi) + (\tau + \gamma_2 c)\phi(\xi)^3 + (\gamma_3 c + \gamma)\phi(\xi)^5 + w^2(1 + 3\gamma_1 c)\phi(\xi)'' = 0 \tag{3}$$

$$(\alpha + 2cw + 3\gamma_1 c^2 w)\phi(\xi)' - 3\gamma_2 w\phi(\xi)^2\phi(\xi)' - 5\gamma_3 w\phi(\xi)^4\phi(\xi)' - \gamma_1 w^3\phi(\xi)''' = 0 \tag{4}$$

Taking $c = -\frac{1}{3\gamma_1}$, in (3) yields

$$-\left(k + \frac{2}{27\gamma_1^2}\right)\phi(\xi) + \left(\tau - \frac{\gamma_2}{3\gamma_1}\right)\phi(\xi)^3 + \left(\gamma - \frac{\gamma_3}{3\gamma_1}\right)\phi(\xi)^5 = 0. \tag{5}$$

Solving (5), we have

$$k = -\frac{2}{27\gamma_1^2}, \quad \tau = \frac{\gamma_2}{3\gamma_1}, \quad \gamma = \frac{\gamma_3}{3\gamma_1} \tag{6}$$

Integrating Eq. (4) twice, we have

$$(\phi'(\xi))^2 = a_0 + \frac{\alpha - \frac{w}{3\gamma_1}}{\gamma_1 w^3}\phi(\xi)^2 - \frac{\gamma_2}{2\gamma_1 w^2}\phi(\xi)^4 - \frac{\gamma_3}{3\gamma_1 w^2}\phi(\xi)^6 \tag{7}$$

where a_0 is a integral constant. Applying generalized auxiliary equation mapping method [7] to Eq. (7), we obtain the following solutions:

Case 1: If we take $a_0 = -\frac{\gamma_2^3}{48\gamma_3^2 w^2 \gamma_1},\ \alpha = -\frac{3w\gamma_2^2}{16\gamma_3} + \frac{w}{3\gamma_1}$, we have

$$(\phi(\xi)')^2 = -\frac{\gamma_2^3}{48\gamma_3^2 w^2\gamma_1} - \frac{3\gamma_2^2}{16\gamma_3\gamma_1 w^2}\phi(\xi)^2 - \frac{\gamma_2}{2\gamma_1 w^2}\phi(\xi)^4 - \frac{\gamma_3}{3\gamma_1 w^2}\phi(\xi)^6 \tag{8}$$

(**1**) when $\gamma_1\gamma_3 > 0,\ \gamma_1\gamma_2 < 0$, we find out the following peakons soliton solutions

$$u_1(x,t) = \exp\left(i\left(\frac{1}{3\gamma_1}x - \frac{2}{27\gamma_1^2}t\right)\right)\left(-\frac{\gamma_2\tanh^2\left(\pm\sqrt{\frac{3\gamma_2^2}{16\gamma_3\gamma_1 w^2}}\left(\left(-\frac{3w\gamma_2^2}{16\gamma_3} + \frac{w}{3\gamma_1}\right)t - wx\right)\right)}{\gamma_3\left(3 + \tanh^2\left(\pm\sqrt{\frac{3\gamma_2^2}{16\gamma_3\gamma_1 w^2}}\left(\left(-\frac{3w\gamma_2^2}{16\gamma_3} + \frac{w}{3\gamma_1}\right)t - wx\right)\right)\right)}\right)^{\frac{1}{2}} \tag{9}$$

$$u_2(x,t) = \exp\left(i\left(\frac{1}{3\gamma_1}x - \frac{2}{27\gamma_1^2}t\right)\right)\left(-\frac{\gamma_2\coth^2\left(\pm\sqrt{\frac{3\gamma_2^2}{16\gamma_3\gamma_1 w^2}}\left(\left(-\frac{3w\gamma_2^2}{16\gamma_3} + \frac{w}{3\gamma_1}\right)t - wx\right)\right)}{\gamma_3\left(3 + \coth^2\left(\pm\sqrt{\frac{3\gamma_2^2}{16\gamma_3\gamma_1 w^2}}\left(\left(-\frac{3w\gamma_2^2}{16\gamma_3} + \frac{w}{3\gamma_1}\right)t - wx\right)\right)\right)}\right)^{\frac{1}{2}} \tag{10}$$

we consider the structure and behavior of this exact solution, in the case of $t \to \infty,\ u(x,t) \to -\frac{\gamma_2}{4\gamma_3}\exp\left(i\left(\frac{1}{3\gamma_1}x - \frac{2}{27\gamma_1^2}t\right)\right)$, this is a periodic wave which shows there is a homoclinic wave for the RKL model.