



More common errors in finding exact solutions of nonlinear differential equations: Part I

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ABSTRACT

In the recent paper by Kudryashov [11] seven common errors in finding exact solutions of nonlinear differential equations were listed and discussed in detail. We indicate two more common errors concerning the similarity (equivalence with respect to point transformations) and linearizability of differential equations and then discuss the first of them. Classes of generalized KdV and mKdV equations with variable coefficients are used in order to clarify our conclusions. We investigate admissible point transformations in classes of generalized KdV equations, obtain the necessary and sufficient conditions of similarity of such equations to the standard KdV and mKdV equations and carried out the exhaustive group classification of a class of variable-coefficient KdV equations. Then a number of recent papers on such equations are commented using the above results. It is shown that exact solutions were constructed in these papers only for equations which are reduced by point transformations to the standard KdV and mKdV equations. Therefore, exact solutions of such equations can be obtained from known solutions of the standard KdV and mKdV equations in an easier way than by direct solving. The same statement is true for other equations which are equivalent to well-known equations with respect to point transformations.

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1. Introduction

Finding exact (resp. closed-form, resp. explicit) solutions plays a significant role in investigation of physically important partial differential equations, especially, nonlinear ones. There exist a number of famous named solutions in different fields of physics. First such solutions were constructed and intensively studied for models describing flows of fluids (e.g., the Navier–Stokes and Euler equations and the boundary layer equation) since it is difficult to neglect nonlinear effects arising in fluid dynamics. These were the Poiseuille flow, the Burgers vortex, Jeffery–Hamel solution for the flow in a wedge shaped region, the von Karman solution for flow over a single rotating disc etc. (see, e.g., the extensive reviews by Berker and by Pukhnachev [2,25]). Simultaneously with penetrating the nonlinear paradigm into other fields of science, the construction of exact solutions became more and more topical and involved nonlinear diffusion and wave equations and systems of such equations, nonlinear Schrödinger equations, general relativity equations etc. Some equations have only a few known exact solutions (e.g., the Ablowitz–Zappetella traveling wave solutions of the Fisher equation), for other equations wide families of exact solutions parameterized with a number of arbitrary constants and functions have been obtained (integrable and

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linearizable equations and equations possessing large Lie symmetry groups). Last decades the scientific activity in this field permanently grows.

Recently [11] Kudryashov has made an excellent review and listed *seven common errors* in finding exact solutions of non-linear differential equations, which appeared in the modern mathematical and physical literature. The above errors relate to correctness, generality and representation of such solutions and methods of their constructions. In spite of its carefulness, the list by Kudryashov does not exhaust all existing common errors in this subject. We indicate two more common errors, continuing numbering of [11] for convenience of further references. These errors usually arise on the very initial step of consideration, namely, under the choice of differential equations for finding exact solutions and related investigations.

Eighth error. Exact solutions are often constructed with no relation to equivalence of differential equations with respect to point (resp. contact, resp. potential, etc.) transformations.

Ninth error. Exact solutions of linearizable differential equations are sometimes found with no relation to the linearizability property.

Mathematical investigations are always carried out up to equivalence relations defined in sets of similar objects. Equivalent objects differ only in properties which are inessential for certain consideration and, therefore, can be identified in certain sense. This rule is extended to mathematical models, arising in applications.

A usual equivalence used for differential equations is that generated by locally nondegenerate point transformations and called *similarity* [19]. Similar systems of differential equations have similar local properties and similar related local objects (local solutions, symmetries, local conservation laws, etc.).

The purpose of this letter is to discuss the eighth error. In fact this error can be considered as a whole family of different *inaccuracies concerning similarity of differential equations*. We distinguish, e.g., the following kinds of inaccuracies.

1. Any differential equation from the class under consideration is similar to the same classical differential equation for which wide multiparametric families of exact solutions were constructed earlier and possibilities for finding new solutions inequivalent to known ones look as problematic.
2. Only differential equations from a subclass of the class under consideration are similar to classical ones and exact solutions are obtained only for such equations. Sometimes constraints singled out the subclass of equations with found particular solutions from the whole class are explicitly presented, sometimes they are implicitly implied or even not indicated in any way.
3. The class of differential equations under consideration can be mapped by a family of point transformations parameterized with arbitrary elements of the class to another class of simpler structure (resp. with less number of arbitrary elements, resp. with simpler transformational properties, etc.) but this possibility is missed.
4. Similarity of differential equations is used for finding exact solutions but the usage is incomplete (resp. improper, resp. incorrect).

There are rather a lot of published papers in which exact solutions were constructed for equations similar to well-known and/or simpler equations without using the similarity. Choosing examples for commenting, we restricted ourselves with quite recent papers published in certain journals and devoted to considerations of equations related to the KdV and mKdV equations. Our choice is justified by the fact that both the equations are the most known and well-investigated nonlinear equations of mathematical physics. They are connected via the Miura transformation. Since these equations are integrable and have nice symmetry properties, wide families of their exact solutions had been found using different powerful methods (the inverse scattering method, Darboux and Bäcklund transformations, the Hirota bilinear method etc.) and collected in a number of papers, textbooks and handbooks (see, e.g., [1,16,17,26,27] and references therein). At the same time, in the commented papers only very particular results on similar equations were obtained.

The illustration of the above list of inaccuracies with generalized KdV and mKdV equations is additionally justified by their own importance in different fields of physics (see, e.g., references in commented papers). Moreover, the existence of similarity transformations between equations which are related to the KdV and mKdV equations is a well-known fact from the late 1960's. A simple point transformation connecting the Gardner and mKdV equations was already presented in [17]. A more nontrivial point transformation between the standard and cylindrical KdV equations was found in [13] (see also [7]). Then the similarity of certain generalizations of KdV and mKdV equations with coefficients depending on t or (t, x) to these equations was investigated in detail (see, e.g., [4,6–8] and references therein). Later the similarity arguments were permanently used under reviewing and refereeing papers similar to commented ones, cf. [15]. Lie and generalized symmetries of such equations were also investigated (see, e.g., [5,28,30]).

Our paper is organized as follows: In the next section we construct a hierarchy of normalized classes of third order $(1+1)$ -dimensional evolution equations, which is related to the examples commented. This gives the complete description of admissible point transformations within such classes. For two wide subclasses of the variable-coefficient KdV and mKdV equations, jointly covering equations from almost all the examples, we derive the necessary and sufficient condition of similarity of such equations to the standard KdV and mKdV equations, respectively. Exhaustive group analysis of a normalized class of variable-coefficient KdV equations is carried out in Section 3. Section 4 is partitioned into two subsections. In Section

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