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Magnetohydrodynamic non-Darcy mixed convection heat transfer from a vertical heated plate embedded in a porous medium with variable porosity

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A B S T R A C T

A numerical model is developed to study magnetohydrodynamics (MHD) mixed convection from a heated vertical plate embedded in a Newtonian fluid saturated sparsely packed porous medium by considering the variation of permeability, porosity and thermal conductivity. The boundary layer flow in the porous medium is governed by Forchheimer–Brinkman extended Darcy model. The conservation equations that govern the problem are reduced to a system of non-linear ordinary differential equations by using similarity transformations. Because of non-linearity, the governing equations are solved numerically. The effects of magnetic field on velocity and temperature distributions are studied in detail by considering uniform permeability (UP) and variable permeability (VP) of the porous medium and the results are discussed graphically. Besides, skin friction and Nusselt number are also computed for various physical parameters governing the problem under consideration. It is found that the inertial parameter has a significant influence in increasing the flow field and the rate of heat transfer for variable permeability case. The important finding of the present work is that the magnetic field has considerable effects on the boundary layer velocity and on the rate of heat transfer for variable permeability of the porous medium. Further, the results obtained under the limiting conditions were found to be in good agreement with the existing ones.

1. Introduction

In recent years, considerable attention has been evinced on the study of boundary layer flow behavior and heat transfer characteristics of Newtonian fluid past a vertical plate embedded in a fluid saturated porous medium in the presence of a magnetic field because of its wide spectrum of applications in engineering processes, especially in the enhanced recovery of petroleum resources, magnetohydrodynamics (MHD) generators, plasma studies, the design of nuclear reactors, geothermal energy extractions, compacted beds for the chemical industry, drying of porous solid, thermal insulation, and so on. There has been renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. The study of transport properties in porous media with magnetic field subject to heat transfer are characterized by highly non-linear coupled partial differential equations. Cheng [1] has provided an extensive review of early works on free convection in porous media. Nakayama and Koyama [2] have obtained the similarity solution to study free convection in the boundary layer adjacent to a

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Nomenclature

Bo	magnetic field factor
C_b	empirical constant of the second-order
C_{f_x}	skin friction coefficient
c_p	specific heat at constant pressure
d, d^*	constant defined in Eqs. (14) and (15)
Ec	Eckert number
f	dimensionless stream function
g	acceleration due to gravity
Gr	local Grashof number
$k(y)$	permeability of the porous medium
k_0	permeability of the porous medium at the edge of the boundary layer
M	magnetic parameter
Nu_x	Nusselt number
p	pressure
Pr	Prandtl number
Re	local Reynolds number
T	temperature of the fluid near the plate
T_∞	ambient temperature ($T_\infty < T_\mathrm{w}$)
T_w	temperature of the plate
u, v	velocity components along x- and y-direction
U_0	free stream velocity
x	axial coordinate
y	normal coordinate

Greek symbols

$\alpha(y)$	thermal diffusivity
α_0	thermal diffusivity at the edge of the boundary layer
α^*	ratio of viscosities
β	coefficient of thermal expansion of the fluid
β^*	local inertial parameter
$\varepsilon(y)$	porosity of the saturated porous medium
ε_0	porosity of the saturated porous medium at the edge of the boundary layer
η	dimensionless similarity variable
κ	thermal conductivity
μ	viscosity of the fluid
$\bar{\mu}$	effective viscosity of the fluid
ν	kinematics viscosity of the fluid
ρ	density of fluid
ψ	stream function
σ	permeability parameter
σ_m	electrical conductivity
σ^*	ratio of thermal conductivity of the solid to the fluid
θ	dimensionless temperature

Subscripts

∞	condition at the free stream
w	condition at the wall

vertical plate immersed in a thermally stratified porous medium. Hsieh et al. [3] have obtained non-similar solution for combined convection from vertical plates in porous media with variable surface temperature or heat flux. Aldoss et al. [4] have studies mixed convection from a vertical plate embedded in a porous medium in the presence of magnetic field. Merkin [5] analyzed the mixed convection boundary layer flow on an impermeable vertical surface embedded in a saturated porous medium. Minto et al. [6] have considered the free convection flow on a vertical surface embedded in a porous medium. Several investigators have studied the effect of the magnetic field on the free convection flow over a semi-infinite vertical plate. Nield [7] studied Darcy flow in an isotropic porous medium with the effect of a magnetic field and the flow in a medium with anisotropic permeability. He found the effect of uniform applied magnetic field is to reduce the effective permeability in the case of boundary layer flow. Elbashbeshy [8] investigated the free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of a magnetic field.