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Switched modified function projective synchronization of hyperchaotic Qi system with uncertain parameters

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ABSTRACT

This work is involved with switched modified function projective synchronization of two identical Qi hyperchaotic systems using adaptive control method. Switched synchronization of chaotic systems in which a state variable of the drive system synchronize with a different state variable of the response system is a promising type of synchronization as it provides greater security in secure communication. Modified function projective synchronization with the unpredictability of scaling functions can enhance security. Recently formulated hyperchaotic Qi system in the hyperchaotic mode has an extremely broad frequency bandwidth of high magnitudes, verifying its unusual random nature and indicating its great potential for some relevant engineering applications such as secure communications. By Lyapunove stability theory, the adaptive control law and the parameter update law are derived to make the state of two chaotic systems modified function projective synchronized. Synchronization under the effect of noise is also considered. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive controllers.

1. Introduction

Chaos synchronization, an important topic in nonlinear science, has been developed and studied extensively in the last few years. Since Pecora and Carrol [1] introduced a method to synchronize two identical systems with different initial conditions, a variety of approaches have been proposed for the synchronization of chaotic systems which include complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], intermittent lag synchronization [5], time scale synchronization [6], intermittent generalized synchronization [7],reduced order synchronization [8–11], projective synchronization [12–14], modified projective synchronization [15,16] and function projective synchronization [17,18].

Recently Du et al. [19] studied a more general form of function projective synchronization called modified function projective synchronization where the drive and response systems are synchronized upto a desired scaling function matrix in Lorenz system by active control method. Sudheer and Sabir [20] reported modified function projective synchronization in hyperchaotic systems using adaptive control method. The unpredictability of scaling functions in modified function projective synchronization can provide additional security in secure communication.

Most of the works mentioned so far involved mainly with low-dimensional chaotic systems with only one positive Lyapunove exponent. Hyperchaotic systems possessing at least two positive Lyapunove exponents have more complex behaviour and abundant dynamics than chaotic systems and are more suitable for some engineering applications such as secure communication. Hence how to realize synchronization of hyperchaotic systems is an interesting and challenging work. Fortu-

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nately, some existing methods of synchronizing low-dimensional chaotic systems like adaptive control, active control, active backstepping control ,sliding mode control methods can be generalized to synchronize hyperchaotic systems [21–27]. In practical situations parameters are probably unknown and may change time to time. Therefore, how to effectively synchronize two hyperchaotic systems with unknown parameters is an important problem for theoretical research and practical applications. Among different methods of synchronizing two hyperchaotic systems, adaptive control method is an effective one for achieving synchronization of hyperchaotic systems with full unknown parameters [28,29].

Recently, Qi et al. mathematically constructed and electronically built the Qi hyperchaotic system [30]. The system has very large positive Lyapunov exponents over a wide parameter region, implying that the system is a truly hyperchaotic giving the system very strong randomness, a high degree of disorder and extremely rich dynamics. More importantly, the frequency spectral analysis demonstrates that the system in the hyperchaotic mode has an extremely broad frequency bandwidth of high magnitudes, verifying its unusual random nature and indicating its great potential for some relevant engineering applications such as secure communications.

Switched synchronization of chaotic systems in which a state variable of the drive system synchronize with a different state variable of the response system is a promising type of synchronization as it provides greater security in secure communication. Recently, Ucar et al. reported multiswitching synchronization of Lorenz chaotic system by active control method [31].

Motivated by the above discussions, in this paper we propose switched modified function projective synchronization scheme for two identical hyperchaotic systems using adaptive control method. To illustrate the effectiveness of the proposed adaptive switched modified function projective synchronization scheme, we study modified function projective synchronization of two identical Qi hyperchaotic systems. To our knowledge, function projective synchronization in hyperchaotic Qi system is not explored. The organization of the rest of this paper is as follows. Section 2 briefly describes Qi hyperchaotic system and Section 3 proposes adaptive control laws and parameter update rules for the switched modified function projective synchronization of two identical Qi hyperchaotic systems. In Section 4, a numerical example is given to demonstrate the effectiveness of the proposed method. Finally some concluding remarks are given in Section 5.

2. System description

Hyperchaotic Qi system is described as [30]

$$\begin{aligned}\dot{x} &= a(y - x) + yz\\ \dot{y} &= b(x + y) - xz\\ \dot{z} &= -cz - fw + xy\\ \dot{w} &= -dw + gz + xy\end{aligned} \tag{1}$$

In Eq. (1), x, y, z and w are state variables a, b, c, d, f and g are the related parameters. The system is hyperchaotic over a wide parameter range. The hyperchaotic attractor of the system when $a = 50, b = 24, c = 13, d = 8, f = 33$ and $g = 30$ is given in (Fig. 1(a) and (b)).

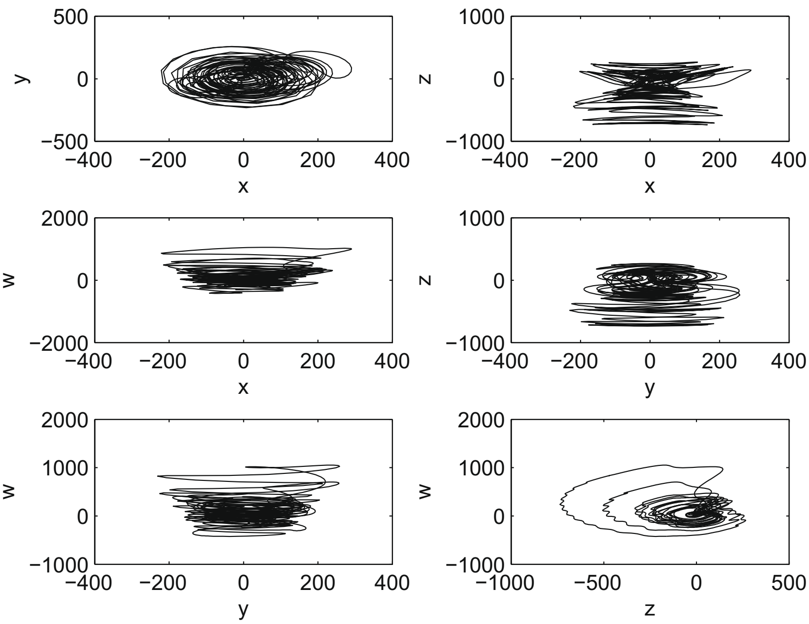

Fig. 1. Chaotic attractor of hyperchaotic Qi system.