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A fast numerical approach to option pricing with stochastic interest rate, stochastic volatility and double jumps

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ABSTRACT

This study proposes a pricing model through allowing for stochastic interest rate and stochastic volatility in the double exponential jump-diffusion setting. The characteristic function of the proposed model is then derived. Fast numerical solutions for European call and put options pricing based on characteristic function and fast Fourier transform (FFT) technique are developed. Simulations show that our numerical technique is accurate, fast and easy to implement, the proposed model is suitable for modeling long-time real-market changes. The model and the proposed option pricing method are useful for empirical analysis of asset returns and risk management in firms.

1. Introduction

Extensive empirical studies suggest that markets tend to have both overreaction and underreaction to various good or bad news [1,2]. One may interpret the jump part of the model as the market response to outside news. Up to now, various aspects of jump-diffusion models have been investigated in the academic finance community [3]. In the last decade, the research departments of major banks also started to accept jump-diffusions as a evaluation tool in their day-to-day modeling. To the best of our knowledge, this increasing interest to jump models in finance is mainly due to the following reasons. On one hand, in a model with continuous paths like a diffusion model, the price process behaves locally like a Brownian motion and the probability that the stock moves by a large amount over a short period of time is very small. Therefore, in such models, the prices of short term out-of-the-money (OTM) options should be much lower than what one observes in real markets. On the other hand, if stock prices are allowed to jump, even when the time to maturity is very short, there is a non-negligible probability that after a sudden change in the stock price the option will move in the money. Furthermore, from a risk management perspective, jumps allow to quantify and take into account the risk of strong stock price movements over short time intervals, which appears non-existent in the diffusion framework.

Two well known jump-diffusion models of Merton [4], with lognormal jumps and Kou [5], with double exponential distributed jumps are both able to partly explain the observed deviations from the Black–Scholes (BS) model [6] which are characterized by the leptokurtic feature and volatility smile. In the Merton's framework, many authors have developed more realistic models, for example, stochastic interest rate models with jumps of Paul Glasserman and Kou [7], Johannes [8] and Bo et al. [9]; stochastic volatility models with jumps of Bates [10] and Pillay and O'Hara [11]. Moreover, several authors

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have developed more general models by combining stochastic interest rate, stochastic volatility and jumps, such as Scott [12], Espinosa and Vives [13] and Jiang [14].

However, in the Kou's framework, the further work is rarely done. Although the double exponential jump-diffusion (DEJD) model has gained wide acceptance because it generates a highly skewed and leptokurtic distribution and is capable of matching key features of stock and index returns. However, as stated by Birge and Linetsky [15], the DEJD model with constant interest rate and constant volatility does not capture all the characteristics of stock returns. For example, it cannot capture the volatility clustering which can be captured by stochastic volatility models. In addition, the spot interest rate is a fundamental economic variable in the economy and it cannot be treated as a constant. Therefore, the model combining stochastic volatility, stochastic interest rate and double exponential jumps may be more reasonable.

In the double exponential jump-diffusion setting, the option price has a complex analytic form [5]. To allow for stochastic volatility and stochastic interest rate will undoubtedly increase the complexity [16,17], a numerical method is thus necessary. Monte Carlo simulation and finite difference method are usually used to value the options [18–21]. But, the time complexity of these two techniques is substantially high, thus they are rarely used in practice. Recently, FFT has been widely used in valuing financial derivatives [22–25], since it is fast, accurate and easy to implement. In this paper we will focus on European option pricing by the FFT technique proposed by Carr and Madan [22]. The use of the FFT is motivated by two reasons. On one hand, the algorithm offers a speed advantage. On the other hand, the characteristic function of the log price is known and has a simple form for many models considered in literature while the density is often unknown in the closed form.

The rest of the paper is organized as follows. Section 2 develops the underlying pricing model. Section 3 derives a closed-form representation of the characteristic function of the proposed model. Section 4 provides fast numerical solutions of European option by FFT technique. Section 5 presents some numerical experiments to examine the effectiveness and efficiency of the proposed model. Section 6 concludes.

2. Allowing for stochastic interest rate and stochastic volatility in DEJD model

Let $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leqslant t\leqslant T}, P\}$ be a complete probability space with a filtration satisfying the usual conditions, i.e., the filtration is continuous on the right and $\mathcal{F}_0$ contains all P-null sets, and P is a risk-neutral probability. Suppose the instantaneous interest rate $r(t)$ is governed by the following square-root mean reversion process

$$dr(t) = (\theta_r - \alpha_r r(t))dt + \sigma_r\sqrt{r(t)}dZ_1(t), \tag{2.1}$$

where $Z_1(t)$ is a Brownian motion which is $\mathcal{F}_t$-adapted. Nonnegative constants $\theta_r, \frac{\theta_r}{\alpha_r}$ and σ_r respectively reflect the speed of adjustment, the long-run mean, and the variation coefficient of $r(t)$, and suppose $r(0) = r$. Because of the affine structure of (2.1) [26], one can express the price of a zero coupon bond maturing at time T into an exponential affine form

$$P(t,T) = A(t,T)\exp[-B(t,T)r(t)], \tag{2.2}$$

where

$$A(t,T) = \left[\frac{2m_1 e^{(\alpha_r+m_1)(T-t)/2}}{(\alpha_r+m_1)(e^{m_1(T-t)}-1)+2m_1}\right]^{\frac{2\theta_r}{\sigma_r^2}},$$
$$B(t,T) = \frac{2(e^{m_1(T-t)}-1)}{(\alpha_r+m_1)(e^{m_1(T-t)}-1)+2m_1},$$
$$m_1 = \sqrt{\alpha_r^2 + 2\sigma_r^2}.$$

Assume that volatility process $V(t)$ is also governed by the square-root mean reversion process

$$dV(t) = (\theta_v - \alpha_v V(t))dt + \sigma_v\sqrt{V(t)}dZ_2(t), \tag{2.3}$$

where $Z_2(t)$ is a Brownian motion which is $\mathcal{F}_t$-adapted. And nonnegative constants $\theta_v, \frac{\theta_v}{\alpha_v}$ and σ_v respectively reflect the speed of adjustment, the long-run mean, and the variation coefficient of $V(t)$, and suppose $V(0) = V$.

Let $S(t)$ be the price for a stock, or a stock portfolio. The asset price process $S(t)$ is governed by the following jump-diffusion process:

$$dS(t) = (r(t) - \lambda\delta)S(t)dt + \sigma\sqrt{V(t)}S(t)dW(t) + S(t)d\left(\sum_{j=1}^{N(t)}(U_j - 1)\right), \tag{2.4}$$

where $W(t)$ is a Brownian motion that is independent of $Z_1(t)$ but correlated with $Z_2(t)$, so that $Cov(dW(t), dZ_2(t)) = \rho dt$. The variation coefficient σ is a nonnegative constant and suppose $S(0) = s$. $N(t)$ is a Poisson process with constant intensity $\lambda > 0,\ \delta = E(U-1)$ and $U = (U_j)_{j\geqslant 1}$ is a sequence of independent and identically distributed nonnegative random variables, such that $Y = \ln U$ has an asymmetric double exponential distribution with the density

$$f_Y(y) = p\eta_1 e^{-\eta_1 y}\mathbf{1}_{y\geqslant 0} + q\eta_2 e^{\eta_2 y}\mathbf{1}_{y<0}, \quad \eta_1 > 1,\ \eta_2 > 0, \tag{2.5}$$