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Review

Stability and Hopf bifurcation analysis on a four-neuron BAM neural network with distributed delays

Baoxian Wang *, Jigui Jian

Institute of Nonlinear Complex Systems, China Three Gorges University, Yichang, Hubei 443002, China

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ABSTRACT

In this paper, a four-neuron BAM neural network with distributed delays is considered, where kernels are chosen as weak kernels. Its dynamics is studied in terms of local stability analysis and Hopf bifurcation analysis. By choosing the average delay as a bifurcation parameter and analyzing the associated characteristic equation, Hopf bifurcation occurs when the bifurcation parameter passes through some exceptive values. The stability of bifurcating periodic solutions and a formula for determining the direction of Hopf bifurcation are determined by applying the normal form theory and the center manifold theorem. Finally, numerical simulation results are given to validate the theorem obtained.

Contents

1. Introduction

In recent years, neural networks such as Hopfield neural networks, cellular neural networks have attracted many scholars' attention all over the world and have been applied in many fields such as image and signal processing, pattern recognition,

* Corresponding author.
E-mail addresses: sxhanmei81@163.com (B. Wang), jianjigui@sohu.com (J. Jian).

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optimization and automatic control and so on. The bi-directional associative memory (BAM) neural networks are peculiar and also have been investigated in past two decades due to its wide applicability. Kosko firstly proposed BAM neural networks and studied the stability of this network described by a system of ordinary differential equation. From then on, academicians have investigated the dynamical behaviors (including stable, unstable, oscillatory, and chaotic behavior). It is celebrated that a neural network model with distributed delay is more general than that with discrete delay, because the distributed delay becomes a discrete delay when the delay kernel is a delta function at a certain time. So investigating dynamical behaviors of neural networks with distributed delays is a significant problem.

There are a great number of papers studying the Hopf bifurcation and the local stability of neural networks with discrete delays [5,8,10,12–15] or distributed delays [1,7,9,11,18]. For example, in [7], authors have studied the Hopf bifurcation of Chen's system with distributed delays. In [18], authors have worked over the local stability and Hopf bifurcation of two-neuron Cohen-Grossberg neural network with distributed delays. In [5], scholars have researched the bifurcation on a simple neural networks with three discrete delays. In [15], authors have investigated the stability and Hopf bifurcation on a four-neuron BAM neural networks with four time delays. Moreover, in [4,6,16,17] authors have already studied the stability and bifurcation on a discrete-time neural network model. Till now, nobody studied the Hopf bifurcation of the BAM neural network with distributed delays. It is inevitable that the complexity of the characteristic equation exists in the analysis of linearized equation because the characteristic equation is usually transcendental (see [2]). Sometimes characteristic equation is a kind of polynomials and the degree of the polynomials is too high to be analyzed. So we only investigate the neural network with four neurons.

Motivated by the above discussion, in the paper, we shall discuss the local stability and Hopf bifurcation on a four-neuron BAM neural network model with distributed delays. Also, we discuss a formula for determining the direction of Hopf bifurcation and the stability of bifurcating periodic solution by using the normal form method and the center manifold theorem introduced by Hassard at [3]. The organization of this paper is as follows: in Section 2, the local stability of trivial solutions and the existence of Hopf bifurcation are discussed. In Section 3, a formula for determining the direction of Hopf bifurcation and the stability of bifurcating periodic solution are derived. Finally, in Section 4, we give some numerical simulations to testify the theoretical analysis.

2. Existence of Hopf bifurcation

The BAM neural network model with distributed delays is described by the following equation group:

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t)) + \sum\limits_{j=1}^{n} w_{ji} \int_0^{+\infty} K_{ji}(\mu) f_j(y_j(t-\mu))d\mu, & i = 1,2,\cdots,m, \\ \dot{y}_j(t) = -b_j(y_j(t)) + \sum\limits_{i=1}^{m} v_{ij} \int_0^{+\infty} L_{ij}(\mu) g_i(x_i(t-\mu))d\mu, & j = 1,2,\cdots,n, \end{cases} \tag{1}$$

here $m \geqslant 2$ is the number of neurons in the I layer and $n \geqslant 2$ is the number of neurons in the J layer in the network; x_i, y_j denote the state variables associated with the neuron, and a_i, b_j are appropriately behaved functions. The connection matrices $W = (w_{ji})_{n\times m}, V = (v_{ij})_{m\times n}$ tell us how the neurons are connected in the network and the activation functions f_j and g_i show how neurons respond to each other. For convenience, we shall discuss the stability and Hopf bifurcation on a four-neuron BAM neural network model with $f_j(y_j(t)) = y_j(t), g_i(x_i(t)) = x_i(t)$, i.e.,

$$\begin{cases} \dot{x}_i(t) = -a_i(x_i(t)) + \sum\limits_{j=1}^{2} w_{ji} \int_0^{+\infty} K_{ji}(\mu) y_j(t-\mu)d\mu, & i = 1,2, \\ \dot{y}_j(t) = -b_j(y_j(t)) + \sum\limits_{i=1}^{2} v_{ij} \int_0^{+\infty} L_{ij}(\mu) x_i(t-\mu)d\mu, & j = 1,2. \end{cases} \tag{2}$$

The initial value conditions are $x_i(s) = \varphi_i(s), y_j(s) = \varphi_{m+j}(s),\ s \in (-\infty, 0]$, where φ_i, and $\varphi_{m+j} (i = 1,2, j = 1,2)$ are continuous and bounded functions. $K_{ji}(\mu), L_{ij}(\mu) (i,j = 1,2)$ are nonnegative bounded functions defined on $[0, +\infty)$ to reflect the influence of the past states or the current dynamics. The presence of the distributed time delays must not affect the equilibrium values, so we normalize the kernels such that $\int_0^{+\infty} K_{ji}(\mu)d\mu = 1, \int_0^{+\infty} L_{ij}(\mu)d\mu = 1 (i,j = 1,2)$ hold.

We define the average time delays as

$$T_{K_{ji}} = \int_0^{+\infty} \mu K_{ji}(\mu)d\mu, T_{L_{ij}} = \int_0^{+\infty} \mu L_{ij}(\mu)d\mu.$$

In particular, in this paper, we take the kernels as the weak kernels

$$K_{ji}(\mu) = \alpha e^{-\alpha\mu}, \quad L_{ij}(\mu) = \alpha e^{-\alpha\mu}, \alpha > 0.$$

Then the average time delays are

$$T_{K_{ji}} = \int_0^{+\infty} \mu\alpha e^{-\alpha\mu} d\mu = \frac{1}{\alpha}, \quad T_{L_{ij}} = \int_0^{+\infty} \mu\alpha e^{-\alpha\mu} d\mu = \frac{1}{\alpha}.$$

To establish the main results for model (2), it is necessary to make the following assumptions

(A_1) Functions $a_i(x_i(t)), b_j(y_j(t))$ are continuous, bounded, positive and satisfy $a_i(0) = 0,\ b_j(0) = 0$ for $i,j = 1,2$.