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A new hyperchaotic system and its circuit implementation

Niu Yujun [a], Wang Xingyuan [a,*], Wang Mingjun [a], Zhang Huaguang [b]

[a] School of Electronic & Information Engineering, Dalian University of Technology, Dalian 116024, China
[b] College of Information Science and Engineering, Northeastern University, Shenyang 110004, China

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ABSTRACT

In this paper, a new hyperchaotic system is presented by adding a nonlinear controller to the three-dimensional autonomous chaotic system. The generated hyperchaotic system undergoes hyperchaos, chaos, and some different periodic orbits with control parameters changed. The complex dynamic behaviors are verified by means of Lyapunov exponent spectrum, bifurcation analysis, phase portraits and circuit realization. The Multisim results of the hyperchaotic circuit were well agreed with the simulation results.

1. Introduction

Hyperchaos was firstly reported by Rössler in 1979 [1], and the first circuit implementation of hyperchaos was realized by Matsumoto et al. [2]. Hyperchaotic system is usually defined as a chaotic system with more than one positive Lyapunov exponent, indicating that the chaotic dynamics of the system are expanded in more than one direction giving rise to a more complex attractor. Hyperchaos has been studied with increasing interest in recent years, in the fields of nonlinear circuits [3], secure communications [4,5], lasers [6], Colpitts oscillators [7], control [8–11], and synchronization [12–15]. Due to its great potential in technological applications, the generation of hyperchaos has become a focal topic for research recently[16–20], in particular purposefully designing a hyperchaotic system from an originally chaotic but non-hyperchaotic system with some simple feedback control techniques, is a theoretically very attractive task. For example, several four-dimensional hyperchaotic systems [16–20] have been found in this way, based on some well-known three-dimensional chaotic system such as on the Chen system [21], the Lü system [22], the generalized Lorenz system [23], a unified chaotic system [24], and so on. This, however, is technically challenging, due to its very complicated hyperchaotic behavior and the lack of a general fundamental theory.

This Letter presents a new hyperchaotic system, which is generated by driving the three-dimensional autonomous chaotic system [25] with a nonlinear controller. The generated hyperchaotic system is not only demonstrated by Lyapunov exponent spectrum, bifurcation analysis, phase portraits but also verified with circuit realization. The Multisim results of the hyperchaotic circuit show very good agreement with the simulation results. For this three-dimensional autonomous chaotic system, we give some discussions on how to build the control law so that hyperchaoticity be observed. It may be the inspiration to the formation and perfection of the general control law.

* Corresponding author.
E-mail address: wangxy@dlut.edu.cn (W. Xingyuan).

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2. Design of new hyperchaotic system

Qi et al. introduced a new three-dimensional autonomous chaotic system [25], which is described as

$$\begin{cases} \dot{x} = a(y - x) + yz \\ \dot{y} = cx - y - xz \\ \dot{z} = xy - bz \end{cases}, \tag{1}$$

where x, y, z are state variables and a, b, c are system parameters. When $a = 35$, $b = 8/3$, $c = 55$, system (1) exhibits a chaotic behavior with a positive Lyapunov exponent. The corresponding chaotic attractor is depicted in Fig. 1. More detailed complex dynamics of system (1) can be seen in Ref. [25].

We know that, in order to obtain hyperchaos, two important requisites are as follows:

(1) The minimal dimension of the phase space that embeds a hyperchaotic attractor should be at least four, which requires the minimum number of coupled first-order autonomous ordinary differential equations to be four.
(2) The number of terms in the coupled equations giving rise to instability should be at least two, of which at least one should have a nonlinear function [1].

Now, by introducing a simple dynamic feedback control term w to the second equation of system (1), the new dynamic system is obtained:

$$\begin{cases} \dot{x} = a(y - x) + yz \\ \dot{y} = cx - y - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -xz + rw \end{cases}, \tag{2}$$

where r is a control parameter, determining the chaotic attractor and bifurcations of system (2). Obviously, the chaotic system (2) is a four-dimensional dynamical system, which has four Lyapunov exponents. It will be shown that this new system can be hyperchaotic with some suitably chosen control parameter r.

To ensure system (2) be dissipative, it is required that

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a - 1 - b + r = r - 38.667 < 0. \tag{3}$$

Therefore, theoretically the largest value of r is 38.667. Choose r near zero so that the previous dissipative structure can be maintained most, according to the method presented by Ramasubramanian and Sriram [26], we obtain when $r = 1.3$, the Lyapunov exponents: $\lambda_1 = 1.4164$, $\lambda_2 = 0.5318$, $\lambda_3 = 0$, $\lambda_4 = -39.1015$. It is obvious that system (2) exhibits a hyperchaotic behavior. The projections of the hyperchaotic attractor are shown in Fig. 2.

3. Bifurcation analysis

Due to the lack of systematic methodology for purposefully designing a hyperchaotic system to date, the following investigation relies on a combination of mathematical analysis and numerical simulations. In system (2), when parameter r varies, several simulations are carried out, and the outcome of chaotic attractors and careful bifurcation analysis is summarized. Assume the Lyapunov exponents of system (2) are λ_1, λ_2, λ_3 and λ_4. We found that:

(1) For periodic orbits, $\lambda_1, \lambda_2, \lambda_3 < 0$, $\lambda_4 = 0$.

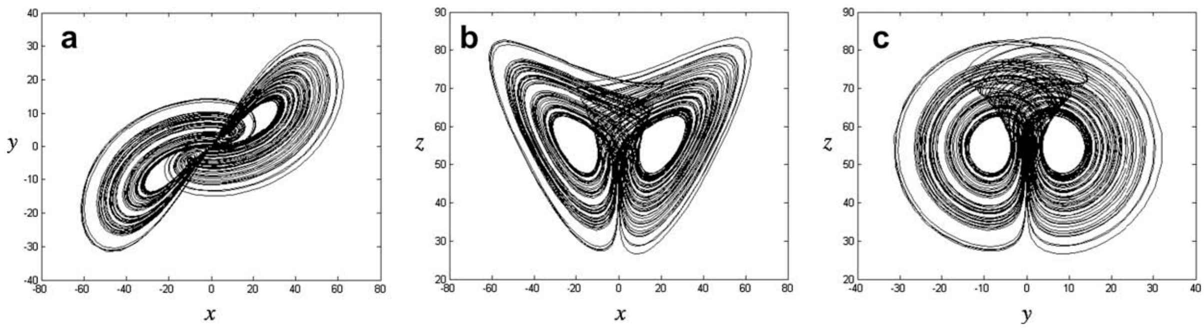

Fig. 1. The projections of attractor system (1).