Commun Nonlinear Sci Numer Simulat 15 (2010) 3599–3606

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Exponential stability of fuzzy Cohen–Grossberg neural networks with time delays and impulsive effects ☆

Changzhao Li [a], Yongkun Li [a], Yuan Ye [b,*]

[a] Department of Mathematics, Yunnan University, Kunming, Yunnan 650091, People's Republic of China
[b] Graduate School of Yunnan University, Yunnan University, Kunming, Yunnan 650091, People's Republic of China

ARTICLE INFO

Article history:
Received 19 April 2009
Received in revised form 31 December 2009
Accepted 1 January 2010
Available online 11 January 2010

Keywords:
Fuzzy Cohen–Grossberg neural networks (FCGNNs)
Global exponential stability
Time delays
Impulsive effects

ABSTRACT

In this paper, we investigate a class of fuzzy Cohen-Grossberg neural networks with time delays and impulsive effects. By employing an inequality technique, we find sufficient conditions for the existence, uniqueness, global exponential stability of the equilibrium without using the *M*-matrix theory. An example is given to illustrate the effectiveness of the obtained results.

1. Introduction

In recent years, the well-known Cohen–Grossberg neural networks [1] have been extensively studied due to their extensive applications in many fields such as pattern recognition, computing associative memory, signal and image processing and so on, see e.g. [2–5]. In these applications, stability of the model is prerequisite.

In reality, it has been realized that significant time delays as a source of instability and bad performance may occur in neural processing and signal transmission, which may lead to some complex dynamic behaviors [6]. Thus, the stability of neural networks with time delays is interesting and many results are proposed for the neural networks with various delays, see [7–11].

However, in mathematical modeling of real world problems, we encounter some inconveniences besides delays, namely, the complexity and the uncertainty or vagueness. Vagueness is opposite to exactness and we argue that it cannot be avoided in the human way of regarding the world. Any attempt to explain an extensive detailed description necessarily leads to using vague concepts since precise description contains abundant number of details. To understand it, we must group them together – and this can hardly be done precisely. A non-substitutable role is here played by natural language. For the sake of taking vagueness into consideration, fuzzy theory is viewed as a more suitable setting. Based on traditional CNNs, Yang and Yang [12] first introduced the fuzzy cellular neural networks (FCNNs), which integrates fuzzy logic into the structure of traditional CNNs and maintains local connectedness among cells. Unlike previous CNNs, FCNN is a very useful paradigm for image processing problems, which has fuzzy logic between its template input and/or output besides the sum of product

☆ This work is supported by the National Natural Sciences Foundation of People's Republic of China under Grant 10971183.
* Corresponding author.
E-mail address: yye_yd@yahoo.com.cn (Y. Ye).

1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2010.01.001

operation. It is a cornerstone in image processing and pattern recognition. Recently, some results on stability and other behaviors have been derived for fuzzy neural networks with or without time delays (see [13–17]).

Nevertheless, besides delay effects, impulsive effects likewise exist in a wide variety of evolutionary processes in which states are changed abruptly at certain moments of time, involving such fields as medicine and biology, economics, mechanics, etc. There are many interesting results about impulsive neural networks, e.g., Ref. [10,14,18–20]. Since impulsive perturbations can affect dynamical behaviors of the system just as time delays, it is meaningful to consider both time delays and impulsive effects of neural networks. It is known to all that Cohen–Grossberg neural network is one of the most popular and typical network models. Some models such as Hopfield-type neural networks, CNNs, BAM-type models are special cases of Cohen–Grossberg neural networks. For more details about Cohen–Grossberg neural networks, one can see e.g. [21,22]. Therefore, it is both theoretical interesting and practically important to study the stability of FCGNNs with time delays and impulsive effects. To the best of our knowledge, FCGNNs with different activation functions are seldom considered and investigated.

Motivated by the above discussion, in this paper, by employing a new inequality, we investigate a class of fuzzy Cohen–Grossberg neural networks with time delays and impulsive effects described by the following system:

$$\begin{cases} \frac{dx_i(t)}{dt} = \alpha_i(x_i(t))\Big[-\beta_i(x_i(t)) + \sum\limits_{j=1}^{n}\gamma_{ij}\mu_j + I_i + \bigwedge\limits_{j=1}^{n} a_{ij}f_j(x_j(t)) + \bigwedge\limits_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_{ij})) \\ \qquad + \bigwedge\limits_{j=1}^{n} T_{ij}\mu_j + \bigvee\limits_{j=1}^{n} c_{ij}f_j(x_j(t)) + \bigvee\limits_{j=1}^{n} d_{ij}g_j(x_j(t-\tau_{ij})) + \bigvee\limits_{j=1}^{n} H_{ij}\mu_j\Big], \quad t\neq t_k, \quad t \geqslant t_0, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \Delta_{ik}(x_i(t_k)), \quad t = t_k, \end{cases} \tag{1.1}$$

for $i = 1,2,\ldots,n;\ k = 1,2,\ldots$, where $x_i(t)$ is the ith neuron state, $\alpha_i(x_i(t))$ represents an amplification function, $\beta_i(x_i(t))$ is an appropriately behaved function, $f_j,\ g_j$ denote the activation functions, τ_{ij} with $0 \leqslant \tau_{ij} \leqslant \tau$ (where $\tau = \max_{1\leqslant i,j\leqslant n}\{\tau_{ij}\}$ is a constant, $i,j \in N$) is the transmission delay, γ_{ij} is the element of fuzzy feed-forward template, $a_{ij},\ b_{ij}$ are elements of fuzzy feedback MIN template, $c_{ij},\ d_{ij}$ are elements of fuzzy feedback MAX template, T_{ij} and H_{ij} are elements of fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operations defined as

$$u \bigwedge v(x) = u(x) \bigwedge v(x) = \min\{u(x), v(x)\}$$

and

$$u \bigvee v(x) = u(x) \bigvee v(x) = \max\{u(x), v(x)\},$$

respectively, where $u,\ v$ are membership functions of x. μ_i and I_i denote input and bias of the ith neuron, respectively. t_k is called impulsive moment and satisfies $0 < t_1 < t_2 < \ldots, \lim_{k\to\infty} t_k = +\infty;\ x_i(t_k^-)$ and $x_i(t_k^+)$ denote the left limit and the right limit at t_k, respectively; $\Delta_k(x(t_k)) = (\Delta_{1k}(x_1(t_k)), \Delta_{2k}(x_2(t_k)), \ldots, \Delta_{nk}(x_n(t_k)))^T, \Delta_{ik}(x_i(t_k))$ shows impulsive perturbation of the ith neuron at t_k.

Remark 1.1. If $\Delta_{ik}(x_i(t_k)) = 0\ (i = 1,2,\ldots,n;\ k = 1,2,\ldots)$, then model (1.1) turns to continuous FCGNN

$$\begin{aligned} \frac{dx_i(t)}{dt} = \alpha_i(x_i(t))\Big[&-\beta_i(x_i(t)) + \sum_{j=1}^{n}\gamma_{ij}\mu_j + I_i + \bigwedge_{j=1}^{n} a_{ij}f_j(x_j(t)) + \bigwedge_{j=1}^{n} b_{ij}g_j(x_j(t-\tau_{ij})) + \bigwedge_{j=1}^{n} T_{ij}\mu_j + \bigvee_{j=1}^{n} c_{ij}f_j(x_j(t)) \\ &+ \bigvee_{j=1}^{n} d_{ij}g_j(x_j(t-\tau_{ij})) + \bigvee_{j=1}^{n} H_{ij}\mu_j\Big], \quad t \geqslant t_0. \end{aligned} \tag{1.2}$$

Throughout this paper, we assume that

(H$_1$) $\alpha_i(u)$ is a continuous function and $0 < \underline{\alpha}_i \leqslant \alpha_i(u) \leqslant \bar{\alpha}_i$ ($\underline{\alpha}_i$ and $\bar{\alpha}_i$ are constants) for all $u \in R$, $i = 1,2,\ldots,n$.

(H$_2$) There exists a positive diagonal matrix $\beta = \mathrm{diag}(\beta_1, \beta_2, \ldots, \beta_n)$ such that

$$\frac{\beta_i(x) - \beta_i(y)}{x - y} \geqslant \beta_i > 0, \quad x\neq y, \quad i = 1,2,\ldots,n.$$

(H$_3$) There exist positive diagonal matrices $L^f = \mathrm{diag}\left(L_1^f, L_2^f, \ldots, L_n^f\right),\ L^g = \mathrm{diag}(L_1^g, L_2^g, \ldots, L_n^g)$ such that

$$L_j^f = \sup_{x\neq y}\left|\frac{f_j(x) - f_j(y)}{x - y}\right|, \quad L_j^g = \sup_{x\neq y}\left|\frac{g_j(x) - g_j(y)}{x - y}\right|, \quad x\neq y, \quad j = 1,2,\ldots,n.$$

(H$_4$) There exist constants $p \geqslant 1$ and $d_i > 0$ $(i = 1,2,\ldots,n)$, such that

$$\beta_i > \frac{p-1}{p}\sum_{j=1}^{n}[(|a_{ij}| + |c_{ij}|)L_j^f + (|b_{ij}| + |d_{ij}|)L_j^g] + \frac{1}{p}\sum_{j=1}^{n}\frac{d_j}{d_i}[(|a_{ji}| + |c_{ji}|)L_i^f + (|b_{ji}| + |d_{ji}|)L_i^g]. \tag{1.3}$$