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Unsteady flow and heat transfer in a thin film of Ostwald–de Waele liquid over a stretching surface

K. Vajravelu [a], K.V. Prasad [b], Chiu-On Ng [c,*]

[a] Department of Mathematics, University of Central Florida, Orlando, FL 32816, USA
[b] Department of Mathematics, Bangalore University, Bangalore 560001, India
[c] Department of Mechanical Engineering, The University of Hong Kong, Hong Kong, People's Republic of China

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ABSTRACT

In this paper, the effects of viscous dissipation and the temperature-dependent thermal conductivity on an unsteady flow and heat transfer in a thin liquid film of a non-Newtonian Ostwald–de Waele fluid over a horizontal porous stretching surface is studied. Using a similarity transformation, the time-dependent boundary-layer equations are reduced to a set of non-linear ordinary differential equations. The resulting five parameter problem is solved by the Keller–Box method. The effects of the unsteady parameter on the film thickness are explored numerically for different values of the power-law index parameter and the injection parameter. Numerical results for the velocity, the temperature, the skin friction and the wall-temperature gradient are presented through graphs and tables for different values of the pertinent parameter. One of the important findings of the study is that the film thickness increases with an increase in the power-law index parameter (as well as the injection parameter). Quite the opposite is true with the unsteady parameter. Furthermore, the wall-temperature gradient decreases with an increase in the Eckert number or the variable thermal conductivity parameter. Furthermore, the surface temperature of a shear thinning fluid is larger compared to the Newtonian and shear thickening fluids. The results obtained reveal many interesting behaviors that warrant further study of the equations related to non-Newtonian fluid phenomena, especially the shear-thinning phenomena.

1. Introduction

During the past two decades, due to its applications to several areas in science and engineering, considerable attention has been devoted to the study of flow and heat transfer within a thin liquid film on an unsteady stretching sheet. These areas include extrusion processes, wire and fiber coating, polymer processing, food stuff processing, design of various heat exchangers and chemical processing equipment, etc. In particular, in melt-spinning processes, the extrudate from the die is generally drawn and simultaneously stretched into a filament or sheet, which is then solidified through rapid quenching or gradual cooling by direct contact with water or chilled metal rolls. In fact, stretching imparts a unidirectional orientation to the extrudate and, as a consequence, the quality of the final product depends considerably on the flow and heat transfer mechanism. Therefore, the analysis of momentum and thermal transport within a thin liquid film on a continuously stretching surface is important for gaining some fundamental understanding of such processes. Motivated by the process of polymer extrusion, in which the extrudate emerges from a narrow slit, Crane [1] examined the Newtonian fluid flow induced by

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* Corresponding author.
E-mail address: cong@hku.hk (C.-O. Ng).

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the stretching of an elastic flat sheet. Subsequently, several extensions related to Crane's [1] flow problem were made for different physical situations (see [2–6]). In these studies [1–6], the boundary layer equation is considered and the boundary conditions are prescribed at the sheet and on the fluid at infinity. Imposition of a similarity transformation reduced the system to a set of ordinary differential equations (ODEs), which was then solved analytically or numerically.

All the above mentioned studies deal with flow and/or heat transfer from a stretching sheet in a fluid medium extending to infinity. However, in real physical situations involving coating processes, one needs to consider the fluid adhering to the stretching sheet as a finite liquid film. Wang [7] was the first to consider such a flow problem with a finite liquid film of a Newtonian fluid over an unsteady stretching sheet. Later, Usha and Sridharan [8] considered a similar problem of axi-symmetric flow in a liquid film. Dandapat et al. [9] investigated the effects of variable fluid properties and thermo-capillarity on the flow and heat transfer in a liquid film on a horizontal stretching sheet. Further, Liu and Andersson [10] explored the work of [7] to study the thermal characteristics of liquid film on an unsteady stretching surface. Abel et al. [11] studied the heat transfer problem for a thin liquid film in the presence of an external magnetic field with viscous dissipation. Nadeem and Awais [12] analyzed the effect of a thin film flow over an unsteady shrinking sheet with variable viscosity. Recently, Aziz et al. [13] addressed the influence of internal heat generation/absorption on the flow and heat transfer in a thin film on an unsteady stretching sheet.

It should be noted that the flow and heat transfer characteristics are affected not only by the velocity and the thermal boundary conditions but also by the physical properties of the liquid-film. Furthermore, the study of non-Newtonian fluid flow on an unsteady stretching surface is important. Although the fluid employed in material processing or protective castings are generally non-Newtonian (example, most of the paints), there has been a very little work done on the flow and heat transfer of a non-Newtonian liquid film over a stretching surface. Among the most popular rheological models of non-Newtonian fluids is the power-law or Ostwald–de Waele model. This model deals with a simple non-linear equation of state for inelastic fluids; this includes linear Newton-fluids as a special case. The power-law model provides an adequate representation of many non-Newtonian fluids for range of shear rates. For instance, Andersson et al. [14] carried out a numerical study for the hydro-dynamical problem of a power-law fluid flow with in a liquid film over a stretching sheet. Here, the thermo-physical properties of the ambient fluid are assumed to be constant. However, it is well known that these properties may change with temperature, especially the thermal conductivity. Available literature [15–17] on variable thermal conductivity shows that this type of work has not been carried out for non-Newtonian fluid obeying the Ostwald–de Waele power-law model.

The purpose of the present study is to explore the effects of thermo-physical property, namely, the variable thermal conductivity and the viscous dissipation on the heat transfer of an incompressible power-law liquid thin film on an unsteady porous stretching surface. In non-Newtonian liquid thin film flow, the effects of variable thermal conductivity, power law index, and viscous dissipation play a significant role in the heat transfer process. Here, the momentum and energy equations are highly non-linear. Hence, a similarity transformation is used to transform the non-linear partial differential equations into nonlinear ordinary differential equations. Due to its complexity and nonlinearity, the proposed problem, is solved numerically by a finite difference scheme known as the Keller box method. The obtained numerical results are used to analyze the flow and heat transfer characteristics of the power-law liquid film that would find applications in manufacturing industries.

2. Formulation of the mathematical problem

Consider an unsteady, two-dimensional, viscous, laminar flow and heat transfer of an incompressible non-Newtonian thin fluid film obeying a power-law model. The flow is due to the stretching of a porous elastic sheet parallel to the x-axis at $y = 0$. Two equal and opposite forces are applied along the x-axis, keeping the origin fixed. A schematic representation of the physical model is presented in Fig. 1. The continuous stretching sheet is assumed to have a prescribed velocity $U_s(x,t)$ and temperature $T_s(x,t)$. Further, a thin liquid film of uniform thickness $h(t)$ rests on the horizontal sheet. With the above assumptions, the equations of conservation of mass, momentum, and energy can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}, \tag{2.2}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\kappa(T)\frac{\partial T}{\partial y}\right)+K\left(\frac{\partial u}{\partial y}\right)^{n+1}, \tag{2.3}$$

where u and v are the velocity components along the x and y directions, respectively; ρ is the density, τ_{xy} is the shear stress. Here, we assume the shear stress as

$$\tau_{xy}=-K\left(-\frac{\partial u}{\partial y}\right)^{n}, \tag{2.4}$$