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Flow distribution and environmental dispersivity in a tidal wetland channel of rectangular cross-section

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ABSTRACT

Presented in this paper is a theoretical analysis on flow distribution and environmental dispersivity for a tidal wetland channel of rectangular cross-section. The analytical solution of velocity distribution for the tidal wetland flow is obtained and illustrated with a limiting case covering the known solution for a steady wetland flow. By use of Aris's method of concentration moments, the environmental dispersivity for a pulsed contaminant emission into the tidal wetland flow is rigorously derived and characterized in terms of dimensionless parameters. The solution is shown to be a generalization of the environmental dispersivity for the corresponding steady wetland flow, taking into account the combined action of periodic oscillation and cross-sectional variation of superficial flow as well as the difference between superficial mass dispersivities in the vertical and lateral directions. For a long time evolution of the contaminant cloud, the environmental dispersivity may approach a stable stage of oscillation with a period equal to the period of the superficial flow. The evolution of environmental dispersivity at the initial stage for the tidal wetland flow is shown not monotonous as it does in the case of the steady wetland flow. It is also found that the period of superficial flow has no impact on the necessary time for the environmental dispersivity to attain the stable stage.

1. Introduction

Wetland worldwide is known to be an essential ecosystem, which provides significant ecological and economic values, for example, biodiversity maintenance, pollutants filtration, flood prevention, coastlines protection, groundwater recharge, as well as raw materials provision [1–3]. For the ecological risk assessment and wastewater treatment linked to wetlands [4–12], flow and environmental dispersion are critical issues for predicting the length and duration of influenced region where pollutant concentration is beyond a water quality standard level.

In hydraulic and environmental studies [13–16], rigorous analytical efforts have been made to reveal the spatial distribution of typical steady wetland flows. For the flow through salt marshes mainly vegetated with emergent vegetation, Lightbody and Nepf presented an expression to predict vertical distribution of longitudinal velocity [17,18]. Based on the general momentum equation for wetland flows, Zeng and Chen, Zeng et al., and Chen et al. derived the velocity profiles for various typical wetland flows [19–21]. To reveal characteristics of distinctively vegetated wetland flows, Chen et al. and Wu et al. presented the velocity distribution for a two-layer wetland flow and a two-zone wetland flow, respectively [22,23]. By dividing a flow zone into four sub-zones, White and Nepf explored the flow structure in a partially vegetated channel [24]. However, these analytical solutions only reflect spatial variation of the flow, and so far almost no specific expression has been

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analytically presented to reveal temporal evolution for unsteady flow through a tidal wetland channel of rectangular cross-section, of which most typical is associated with the tidal effect.

Regarding environmental dispersion in the vegetated wetland flows, there have been some analytical investigations, with focus on the impacts of vertical, lateral, and cross-sectional variation of the flow. Lightbody and Nepf presented an expression for total longitudinal dispersion coefficient due to wake-shear at stem-scale and vertical variation of longitudinal velocity at depth-scale in salt marshes [17,18]. Based on Taylor's analysis on dispersion [25] and Aris's method of concentration moments [26], as widely applied to environmental flows in open channels, rivers, and estuaries [27–29], Zeng and Chen, Zeng et al., and Chen et al. derived the environmental dispersivities for various typical flows through single-zone wetlands [19–21], and some of those results were also obtained by Wu et al. with multi-scale analysis [30,31]. For the wetland flow dominated by submerged vegetation, Murphy et al. and Nepf et al. explored the longitudinal dispersion coefficients due to turbulence of different scale, which accounted for the effect of exchange zone between the free stream above a canopy and the flow in the canopy on dispersion [32,33]. Due to the variation of vegetation species composition and distribution in lateral and vertical directions, momentum and mass transport can change greatly in the wetland channel, thereby resulting in large differences of environmental dispersion between single-zone and multi-zone wetland flows, as well as between single-layer and multi-layer wetland flows. Recently, some analytical solutions were presented to predict the environmental dispersivities in a distinctively vegetated two-zone wetland flow and a two-layer wetland flow by Wu et al. and Chen et al., respectively [22,23]. However, these investigations only take the behavior of environmental dispersion contributed by the spatial variation of superficial flow into account, and the effect of unsteadiness of superficial flow on environmental dispersivity has not been considered.

Presented in this paper is an analytical study on flow distribution and environmental dispersivity for a tidal wetland channel of rectangular cross-section. For a pulsed contaminant emission into a tidal flow through a wetland channel, the environmental dispersion due to the combined action of periodic oscillation, cross-sectional variation of superficial flow, as well as the superficial mass dispersion in the vertical and lateral directions is analyzed with Aris's well known concentration moment method [26,34–36].

The specific objectives of this work are: (I) to present a velocity distribution of tidal flow through a wetland channel of rectangular cross-section; (II) to find a concrete expression for environmental dispersivity; (III) to determine the necessary time for the environmental dispersivity to attain a stationary stage, and (IV) to illustrate the effects of dimensionless parameters on the velocity distribution and environmental dispersivity.

2. Formulation

For a typical wetland flow, the governing equations for momentum and mass transport can be written at the phase average scale as follows [20,21,23,37–40]

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t}+\nabla\cdot\frac{\mathbf{U}\mathbf{U}}{\phi}\right)=-\nabla P-\mu F\mathbf{U}+\kappa\mu\nabla^2\mathbf{U}+\kappa\nabla\cdot(\mathbf{L}\cdot\nabla\mathbf{U}), \tag{1}$$

$$\phi\frac{\partial C}{\partial t}+\nabla\cdot(\mathbf{U}C)=\nabla\cdot(\kappa\lambda\phi\nabla C)+\kappa\nabla\cdot(\mathbf{K}\cdot\nabla C), \tag{2}$$

where ρ denotes the density of water, $\mathbf{U}$ velocity, t time, ϕ porosity, P effective pressure including gravitational effect, μ dynamic viscosity, F shear factor, κ tortuosity, $\mathbf{L}$ momentum dispersivity tensor, C concentration, λ mass diffusivity, and $\mathbf{K}$ mass dispersivity tensor.

Consider contaminant transport in a flow with constant ϕ, F, κ, $\mathbf{L}$ and $\mathbf{K}$ through a rectangular wetland channel of width W and depth H, in a Cartesian coordinate system with longitudinal x-coordinate aligned with the flow direction, lateral y-coordinate perpendicular to the bank, vertical z-coordinate towards the free water surface, and origin set at the bank wall, as shown in Fig. 1. $\mathbf{L}$ and $\mathbf{K}$ can be expressed as $\mathbf{L}=L_{xx}\mathbf{e}_x\mathbf{e}_x+L_{yy}\mathbf{e}_y\mathbf{e}_y+L_{zz}\mathbf{e}_z\mathbf{e}_z$ and $\mathbf{K}=K_{xx}\mathbf{e}_x\mathbf{e}_x+K_{yy}\mathbf{e}_y\mathbf{e}_y+K_{zz}\mathbf{e}_z\mathbf{e}_z$, respectively, where L_{xx}, L_{yy}, and L_{zz} are the components of the momentum dispersivity, K_{xx}, K_{yy}, and K_{zz} are the components of the mass dispersivity, and $\mathbf{e}_x, \mathbf{e}_y$, and $\mathbf{e}_z$ are the base vectors in the direction of x, y, and z, respectively [21].

For the case of periodic pressure gradient independent of x, y, and z, Eq. (1) is reduced to

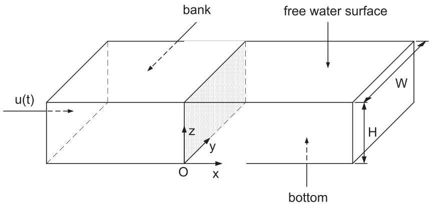

Fig. 1. Sketch for a tidal flow through a wetland channel of rectangular cross-section.