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Complete controllability of fractional evolution systems ☆

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ABSTRACT

The paper is concerned with the complete controllability of fractional evolution systems without involving the compactness of characteristic solution operators introduced by us. The main techniques rely on the fractional calculus, properties of characteristic solution operators and fixed point theorems. Since we do not assume the characteristic solution operators are compact, our theorems guarantee the effectiveness of controllability results in the infinite dimensional spaces.

1. Introduction

In 1960, Kalman first introduced the concept of controllability which leads to some very important conclusions regarding the behavior of linear and nonlinear dynamical systems. There are various work of complete controllability of systems represented by differential equations, integrodifferential equations, differential inclusions, neutral functional differential equations and impulsive differential inclusions in Banach spaces (see [1–13]). Although the complete controllability of integer evolution systems in the infinite dimensional spaces have been discussed extensively, Hernández and O'Regan [14] point out that some papers on controllability of abstract control systems contain a similar technical error when the compactness of semigroup and other hypotheses are satisfied, more precisely, in this case the application of controllability results are restricted to the finite dimensional space. Meanwhile, Ji et al. [15] find some conditions guaranteeing the controllability of impulsive differential systems when the Banach space is nonseparable and evolution systems is not compact, by means of Möch fixed point theorem and the measure of noncompactness.

Recently, the theory of fractional differential equations have become an active area of investigation due to their applications in the fields of physics. There have been a great deal of interest in the solutions of fractional differential equations in analytical and numerical sense. One can see the monographs of Kilbas et al. [16], Miller and Ross [17], Podlubny [18], Lakshmikantham et al. [19], Tarasov [20] and the survey of Agarwal et al. [21]. In order to study the fractional systems in the infinite dimensional spaces, the first important step is how to introduce a new concept of mild solutions. A pioneering work has been reported by El-Borai [22,23] and Zhou and Jiao [24,25]. Particular, Wang et al. [26–28] studied the existence of mild solutions for some semilinear evolution equations and obtained the existence of optimal controls.

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More and more researchers [29–34] pay attention to study the controllability of fractional evolution systems. By applying the similar methods and compactness conditions which inspired by the corresponding integer evolution systems, some interesting controllability results are obtained. Unfortunately, Hernández et al. [35] point that the concept of mild solutions used in [29–31], which inspired by Jaradat et al. [36] was not suitable for fractional evolution systems at all and the corresponding formula of mild solutions is just a simple extension of the mild solutions of integer evolution systems. As a result, it is necessary to restudy this interesting and hot topic again.

Here, we investigate the complete controllability of fractional semilinear systems in the infinite dimensional spaces of the type

$$\begin{cases} {}^{C}D_t^q x(t) = Ax(t) + f(t, x(t)) + Bu(t), \quad t \in J = [0, b], \\ x(0) = x_0 \in X. \end{cases} \tag{1}$$

where ${}^{C}D_t^q$ is the Caputo fractional derivative of order $0 < q \leqslant 1$ with the lower limit zero, for more details on fractional calculus, the reader can refer to [16]. $b > 0$ is a finite number, the state $x(\cdot)$ takes values in a Banach space X and the control function $u(\cdot)$ is given in $L^2(J, U)$, with U as a Banach space. Here, A is the infinitesimal generator of a strongly continuous semigroup $\{T(t), t \geqslant 0\}$ in X, B is a bounded linear operator from U into X, and $f : J \times X \to X$ is given X-valued functions.

In the present paper, we will introduce a suitable concept for mild solutions and establish some sufficient conditions for the complete controllability of system (1) when the operator $T(t),\ t > 0$ is not compact, by means of fractional calculus and Krasnoselskii's fixed point theorem. Since the formula of the mild solutions (see Definition 2.1) including characteristic solution operators $\mathcal{T}(\cdot)$ and $\mathcal{S}(\cdot)$ which are associated with operators semigroup $\{T(t),\ t \geqslant 0\}$ and some probability density functions ξ_q, it is much different from the case of integer order systems. To obtain the complete controllability of system (1) in the infinite dimensional spaces, we put some necessary conditions on $\mathcal{S}(\cdot)$ instead of assuming the semigroup $\{T(t),\ t \geqslant 0\}$ is compact via the strongly continuous property of $\mathcal{T}(\cdot)$.

The rest of this paper is organized as follows. In Section 2, we give some preliminaries and introduce the mild solutions of system (1). In Section 3, the complete controllability results for system (1) are given. At last, an example is provided to illustrate the theory.

2. Preliminaries

We denote by X a Banach space with the norm $\|\cdot\|$ and $A : D(A) \to X$ is the infinitesimal generator of a strongly continuous semigroup $\{T(t),\ t \geqslant 0\}$. This means that there exists $M_1 \geqslant 1$ such that $\sup_{t\in J}\|T(t)\| \leqslant M_1$. Let Y be another Banach space, $L_b(X, Y)$ denote the space of bounded linear operators from X to Y. We also use $\|f\|_{L^p(J,R^+)}$ to denote the $L^p(J, R^+)$ norm of f whenever $f \in L^p(J, R^+)$ for some p with $1 \leqslant p < \infty$. Let $L^p(J, X)$ denote the Banach space of functions f: $J \to X$ which are Bochner integrable normed by $\|f\|_{L^p(J,X)}$. We denote by $\mathcal{C}$, the Banach space $C(J, X)$ endowed with supnorm given by $\|x\|_{\mathcal{C}} \equiv \sup_{t\in J}\|x(t)\|$, for $x \in \mathcal{C}$.

We firstly recall the definition of mild solutions for our problem.

Definition 2.1. For each $u \in L^2(J, U)$, a mild solution of the system (1) we mean the function $x \in \mathcal{C}$ which satisfies

$$x(t) = \mathcal{T}(t)x_0 + \int_0^t (t-s)^{q-1}\mathcal{S}(t-s)f(s, x(s))ds + \int_0^t (t-s)^{q-1}\mathcal{S}(t-s)Bu(s)ds,$$

where $\mathcal{T}(\cdot)$ and $\mathcal{S}(\cdot)$ are called characteristic solution operators and given by

$$\mathcal{T}(t) = \int_0^\infty \xi_q(\theta)T(t^q\theta)d\theta, \quad \mathcal{S}(t) = q\int_0^\infty \theta\xi_q(\theta)T(t^q\theta)d\theta$$

and for $\theta \in (0, \infty)$,

$$\xi_q(\theta) = \frac{1}{q}\theta^{-1-\frac{1}{q}}\varpi_q(\theta^{-\frac{1}{q}}) \geqslant 0,$$
$$\varpi_q(\theta) = \frac{1}{\pi}\sum_{n=1}^{\infty}(-1)^{n-1}\theta^{-qn-1}\frac{\Gamma(nq+1)}{n!}\sin(n\pi q).$$

Here, ξ_q is a probability density function defined on $(0, \infty)$, that is

$$\xi_q(\theta) \geqslant 0, \quad \theta \in (0, \infty) \quad \text{and} \quad \int_0^\infty \xi_q(\theta)d\theta = 1.$$

The following results of $\mathcal{T}(\cdot)$ and $\mathcal{S}(\cdot)$ will be used throughout this paper.

Lemma 2.2 (Lemmas 3.2 and 3.3, [24]). *The operators $\mathcal{T}$ and $\mathcal{S}$ have the following properties:*

(i) *For any fixed $t \geqslant 0$, $\mathcal{T}(t)$ and $\mathcal{S}(t)$ are linear and bounded operators, i.e., for any $x \in X$,*