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Nonlinear dynamics of unicycles in leader–follower formation

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ABSTRACT

In this paper, a dynamical systems analysis is presented for characterizing the motion of a group of unicycles in leader–follower formation. The equilibrium formations are characterized along with their local stability analysis. It is demonstrated that with the variation in control gain, the collective dynamics might undergo Andronov–Hopf and Fold–Hopf bifurcations. The vigor of quasi-periodicity in the regime of Andronov–Hopf bifurcation and heteroclinic bursts between quasi-periodic and chaotic behavior in the regime of Fold–Hopf bifurcation increases with the number of unicycles. Numerical simulations also suggest the occurrence of global bifurcations involving the destruction of heteroclinic orbit.

1. Introduction

Equilibrium formations for nonholonomic systems have been an active area of research in recent times among many disciplines like biological sciences [1–3], computer graphics [4] and systems engineering [5–8]. One particular problem studied in this context has been the *consensus seeking* [9] or the *state agreement problem* [10] which deals with designing feedback controllers to make multiple agents converge to a common configuration in the global coordinates. A special case to this is the *rendezvous problem* [11,12] where the agents converge at a single location.

In addition to the stability and control aspects, considerable efforts have also been put in effective modeling of the nonholonomic systems to make the analysis tractable. Starting from the n-bug problem in mathematics [13], the self-propelled planar particles were later [14,15] replaced by wheeled mobile agents with single nonholonomic constraint, i.e. unicycles. Lie group formulation [16] and oscillator models [17] have been attempted for dynamic modeling of such agents. In particular, Klein and Morgansen [18] extended the oscillator model to account for the intermediate centroid velocity of the unicycles to make trajectory tracking possible.

Several researchers [14,15,19–21] proposed laws for designing control strategies of such nonholonomic vehicles. One possible approach to design the control law is to use a centralized cooperative control scheme for the entire agent collective. However, such a control law is susceptible to bandwidth limitation as well as external disturbances and hence not scalable for a team having large number of mobile agents. As a result, distributed control laws have been investigated by the researchers for this problem, where the feedback is constructed through local interactions of the vehicles leading to a global formation convergence. In particular, Yang et al. [22,23] proposed a decentralized framework where a distributed controller accounts for local control decision based on the interaction of each agent with its neighbors.

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Moreover, their algorithm was also capable of estimating the global statistics of the swarm (for example, overall swarm shape), thereby enabling simultaneous estimation and control. A special research topic has been to design the distributed controller with asynchronous communication constraints. For a detailed account on this topic, the reader may refer [24–27].

The present paper is part of a research endeavor which aims to address the nonholonomic multi-agent dynamics and distributed control problem. The authors earlier studied [28] the cyclic pursuit of 2-unicycle problem with a controller similar to [14] in modified form. These preliminary results showed that the system may exhibit very different dynamics depending on the choice of controller gains. As a next step, in this paper, the authors study nonlinear dynamics of multiple nonholonomic unicycles in leader–follower configuration to characterize regimes of linear stability. Nonlinear analysis is also performed, which throws light in many nontrivial areas of the complex dynamics of the agents leading to greater understanding of the overall system.

The differences in the recent research directions in multi-agent systems has generally varied with the variety of control strategies and the types of consensus demanded. To the best of the authors' knowledge, very few attempts (like [29]) have been made to understand the system from the standpoint of nonlinear dynamics. The authors must underline the fact that a successful analysis to even slightly simpler systems like leader–follower configuration, can guide us in better designing of controllers.

As mentioned above, the choice of leader–follower configuration was partly due to its slightly simpler dynamics and partly due to the fact that many biological systems (like birds) also exhibit this configuration. This choice, in the biological world was long believed to be for energy efficiency [30]. Some recent results [31] tell that leader–follower configuration may also enhance communication and orientation of the flock. It is a topic of research whether this form may have any superiority in inter-agent communication and performance for the bio-mimetic collectives.

The rest of this paper is organized as follows. Section 2 describes the mathematical model of the leader–follower formation and transforms the equations of motion from global to relative coordinates. Section 3 provides the derivation of fixed points followed by corresponding equilibrium formations. Section 4 presents the stability boundary based on local stability analysis and associated Hurwitz stability criteria. Section 5 presents the existence of Andronov–Hopf bifurcation depending on the value of scaled control gain followed by numerical simulation results presented in Section 6. Section 7 concludes the paper.

2. Mathematical model

The focus of this paper is to investigate the dynamics of an n-unicycle system with kinematic equations given by

$$\begin{pmatrix} \dot{x}_j(t) \\ \dot{y}_j(t) \\ \dot{\theta}_j(t) \end{pmatrix} = \begin{pmatrix} \cos\theta_j(t) & 0 \\ \sin\theta_j(t) & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \upsilon_j \\ \omega_j \end{pmatrix}, \quad j = 0, 1, \ldots, n-1, \tag{1}$$

where the position and orientation of the jth vehicle are denoted by $x_j, y_j \in \mathbb{R}$ and $\theta_j \in [-\pi, \pi)$, respectively. υ_j and ω_j are the control inputs (linear velocity and angular velocity). The vehicle with index 0 will be referred to as the *leader*, and the others as *followers*. Our focus here is to analyze the leader–follower formation, when the trajectory of the leader is a straight line or a circle. These trajectories for the leader are obtained by the simple control law

$$\begin{aligned} \upsilon_0 &= V, \\ \omega_0 &= \omega. \end{aligned} \tag{2}$$

The case $\omega = 0$ represents straight line motion, while $\omega \neq 0$ corresponds to circular motion.

The configuration of the n-unicycle system is shown in Fig. 1, where r_i is the relative distance between the two vehicles, α_i is the angle between the current orientation of the ith unicycle and the line of sight, and β_i is the angle between the current orientation of $i+1$th unicycle and the line of sight. Both angles are positive in the sense of counterclockwise rotation to the line of sight. Following [14], the kinematic equations are written in relative coordinates:

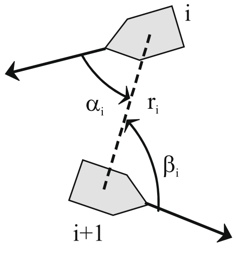

Fig. 1. Relative coordinates with vehicle $i+1$ pursuing vehicle i.