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Shrinking flow of second grade fluid in a rotating frame: An analytic solution

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ABSTRACT

In this work, the homotopy analysis method (HAM) is employed to develop a series solution for shrinking flow in a rotating frame of reference. An incompressible and homogeneous second grade fluid is bounded between the two porous walls. Convergence of the obtained analytic solution is carefully checked. Graphical results are presented and discussed. It is found that the magnitude of x and z-components of dimensionless velocity in viscous fluid is more in comparison to second grade fluid. However the magnitude of dimensionless y-component in second grade fluid is much than that of viscous fluid when $\alpha \leqslant 0.5$.

1. Introduction

Since 1961, attention has been focused to investigate the stretching flows under varied conditions. The boundary layer flow by a continuously solid and moving surface is first performed by Sakiadis [1]. The majority of attempts of linear and nonlinear stretching flows include the viscous fluid model and less is presented for non-Newtonian fluids. Few investigations describing the stretching flows may be mentioned through Refs. [2–14]. Existing literature shows that shrinking flows have been scarcely discussed. Wang [15] obtained the unsteady shrinking film solution. The existence and uniqueness of steady shrinking flow of a viscous fluid for specific suction parameter is studied by Miklavcic and Wang [16]. Sajid et al. [17] examined the steady shrinking flow of a viscous fluid.

In recent time the magnetohydrodynamic flows of non-Newtonian fluids have attracted the attention of engineers, physicists, numerical simulists, modelers and mathematicians as well. These fluids are encountered mainly in industry and technology. The constitutive equations of such fluids give rise to equations which in general are higher order and more complicated than the Navier–Stokes equations. Therefore, one needs the extra boundary/initial conditions for a unique solution. In view of all these challenges, the present paper deals with the steady rotating flow of a shrinking surface. Modeling of the problem is based upon the constitutive equations obeying the magnetohydrodynamic (MHD) second grade fluid in a porous channel. The model of second grade fluid can describe the normal stress effects. However this model does not predict shear thinning/shear thickening, relaxation and retardation effects. Such fluids have promising applications in the petroleum industry, polymer technology, designing cooling systems with liquid metals, MHD generators, flow meters, pumps and in the purification of crude oil etc. The flows in porous channel are particularly important in lubrication and viscometry. The flows under boundary layer approximation are of great value in reducing frictional drag on the hulls of ships and submarines. Moreover these have applications in biomedical engineering, for instance in the dialysis of blood in artificial kidney, blood

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flow in the capillaries, flow in blood oxygenators. Engineering applications include the design of filters, the porous pipe design, in transpiration cooling boundary layer control and gaseous diffusion. The organization of the paper is given as follows.

Section 2 includes the problem development. Series solution is evaluated by HAM [18–35]. Section 4 comprises the convergence of derived solution. Discussion of results is made in Section 5. Section 6 synthesis the main points of the presented analysis.

2. Problem formulation

We investigate the magnetohydrodynamic (MHD) steady flow of a thermodynamic second grade fluid between two porous plates distant $2h$ apart. The lower and upper plates correspond to suction and blowing respectively. Under suction phenomenon, the eventual state of boundary layer is of uniform thickness. Both fluid and plates are rotating with constant angular velocity $\mathbf{\Omega}$ about the y-axis. Fig. 1 shows the geometry of the problem. The appropriate definition of velocity is

$$\mathbf{V} = [u(x,y),\, v(y),\, w(x,y)], \tag{1}$$

where u, v and w are the velocity components in x, y and z-directions, respectively.

The equation of motion for MHD flow in a rotating frame are [36]

$$\rho\left[\frac{d\mathbf{V}}{dt} + 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})\right] = \operatorname{div}\mathbf{T} + \mathbf{J}\times\mathbf{B}, \tag{2}$$

in which $\mathbf{J}$ is the current density and $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}$ is total magnetic field, ρ is the fluid density, d/dt is the material derivative, $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{r}$ is the radial coordinate, σ is the electrical conductivity and $\mathbf{B}_0$ is an applied magnetic field. Note that an induced magnetic field $\mathbf{b}$ is neglected under the assumption of small magnetic Reynolds number. No electric filed is applied. The Cauchy stress tensor $\mathbf{T}$ in a second grade fluid is of the form [7]

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{3}$$

$$\mathbf{A}_1 = \mathbf{L} + \mathbf{L}^T, \tag{4}$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1\mathbf{L} + \mathbf{L}^T\mathbf{A}_1, \tag{5}$$

$$\mathbf{L} = \nabla\mathbf{V}. \tag{6}$$

Here p is the pressure, $\mathbf{I}$ is an identity tensor, μ is the dynamic viscosity, $\alpha_1,\ \alpha_2$ are the material constants and $\mathbf{A}_1,\ \mathbf{A}_2$ are the first two Rivilin Erickson tensors. Furthermore, α_1 and α_2 satisfy the following constraints [37]:

$$\mu \geqslant 0, \quad \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0.$$

From Eqs. (1)–(6) and continuity equation, the resulting equations are given below

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} + 2\Omega w = -\frac{1}{\rho}\frac{\partial p^*}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \frac{\alpha_1}{\rho}\begin{bmatrix} 2\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} + 2u\frac{\partial^3 u}{\partial x^3} \\ +2v\frac{\partial^3 u}{\partial x^2\partial y} + 2\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} + u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} \\ +\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 w}{\partial x\partial y}\frac{\partial w}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} \end{bmatrix} - \frac{\sigma B_0^2 u}{\rho}, \tag{8}$$

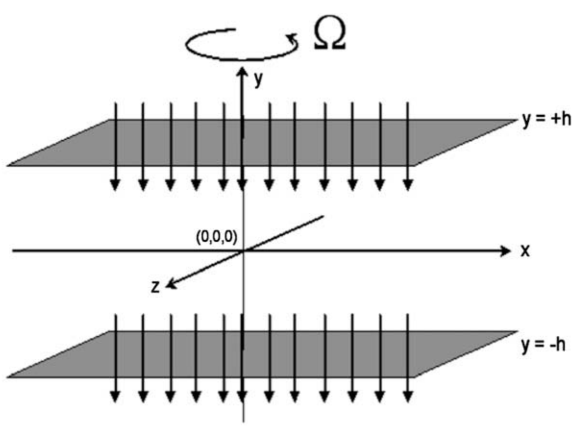

Fig. 1. Schematic diagram of the problem.