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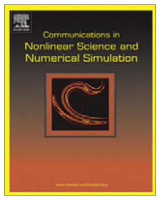

Chaos prediction and control in MEMS resonators

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ABSTRACT

The chaotic dynamics of a micro mechanical resonator with electrostatic forces on both sides is investigated. Using the Melnikov function, an analytical criterion for homoclinic chaos in the form of an inequality is written in terms of the system parameters. Detailed numerical studies including phase portrait, Poincare map and bifurcation diagram confirm the analytical prediction and reveal the effect of excitation amplitude on the system transition to chaos. Moreover a robust adaptive fuzzy control algorithm previously proposed by the authors is applied for controlling the chaotic motion. Additional numerical simulations show the effectiveness of the proposed control approach.

1. Introduction

In many cases it is highly desirable to reduce the size of MEMS mechanical elements [1]. This allows increasing the frequencies of mechanical resonances and improving their sensitivity as sensors. Although miniaturized MEMS resonant systems have many attractions, they also present several challenges. A main problem is how to achieve high output energy. This is an important practical issue for devices such as resonators and micro-sensors. A common solution to the problem is the well-known electrostatic comb-drive [2]. However, this solution adds new constraints to the design of the mechanical structure due to the many complex and undesirable dynamical behaviors associated with it. Another way to face this challenge is to use a strong exciting force [3,4].The major drawback of this approach is the nonlinear effect of the electrostatic force. When a beam is oscillating between parallel electrodes the change in capacitance is not a perfectly linear function.

The forces attempting to restore the beam to its neutral position vary as the beam bends; the more the beam is deflected, the more nonlinearity can be observed. In fact nonlinearities in MEMS resonators generally arise from two distinct sources: relatively large structural deformations and displacement-dependent excitations. Further increasing the magnitude of the excitation force will result in nonlinear vibration of the system which will be shown to be more effective on dynamic behavior of resonator. A great deal of research has been performed to report various nonlinear dynamic phenomena such as bending of the frequency–response curve and jump phenomenon in MEMS resonators [5–7]. Nonlinearities also may lead to chaotic behaviors [8]. Modeling work [9] predicted the existence of chaotic motion in electrostatic MEMS. In [10] the chaotic motion of MEMS resonant systems in the vicinity of specific resonant separatrix is investigated based on corresponding resonant condition. An optimal linear feedback control strategy is used in [11] to reducing the chaotic movement of the system proposed in [10] to a stable orbit. In [12] chaotic behavior of micro-electromechanical oscillator which was modeled by a version of Mathieu equation investigated both numerically

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and experimentally. Chaotic motion was also reported in [13] for a micro-electromechanical cantilever beam under both open and close loop control.

The present paper consists of six sections and one appendix. After a brief introduction in the present section (Section 1) we will examine the mathematical model of micro beam resonator which is excited between two parallel electrodes in Section 2. Melnikov integral is introduced in Section 3. In Section 4 to verify analytical findings and to further explore the chaotic behavior of the resonator several numerical studies are performed. Various fuzzy control methods for control of vibrations are proposed in the literature e.g., [14,15,16]. A robust adaptive fuzzy method is used to control chaotic motion based on [17] in section 5. In the last section (Section 6) we will provide a summary and conclusions. In the appendix integration procedure for the Melnikov function is described.

2. Mathematical model

The electrostatically actuated microbeam is shown in Fig. 1. An external driving force on the resonator is applied via an electrical driving voltage that causes electrostatic excitation with dc-bias voltage between electrodes and the resonator: $V_i = V_b + V_{AC}\mathrm{Sin}\Omega t$ where V_b is the bias voltage, and V_{AC} and Ω are the AC amplitude and frequency, respectively. The net actuation force, F_{act}, can be expressed as [5]:

$$F_{act} = \frac{1}{2}\frac{C_0}{(d-z)^2}(V_b + V_{AC}\mathrm{Sin}\Omega t)^2 - \frac{1}{2}\frac{C_0}{(d+z)^2}V_b^2 \tag{1}$$

where C_0 implies the capacitance of the parallel-plate actuator at rest, d is the initial gap width and z is the vertical displacement of the beam. The governing equation of motion for the dynamics of the MEMS resonator is:

$$m_{eff}z'' + bz' + k_1 z + k_3 z^3 = F_{act} \tag{2}$$

z' and z'' represent the first and second time derivative of z, and m_{eff}, b, $k_1$and k_3 are effective lumped mass, damping coefficient, linear mechanical stiffness and cubic mechanical stiffness of the system respectively. It is convenient to introduce the following dimensionless variables:

$$\tau = \omega_0 t, \quad \omega = \frac{\Omega}{\omega_0}, \quad x = \frac{z}{d}, \quad \mu = \frac{b}{m_{eff}\omega_0}, \quad \alpha = \frac{k_1}{m_{eff}\omega_0^2}, \quad \beta = \frac{k_3 d^2}{m_{eff}\omega_0^2}, \quad \gamma = \frac{C_0 V_b^2}{2m_{eff}\omega_0^2 d^3}, \quad A = 2\gamma\frac{V_{AC}}{V_b} \tag{3}$$

where ω_0 is the purely elastic natural frequency defined as:

$$\omega_0 = \sqrt{\frac{k_1}{m_{eff}}} \tag{4}$$

Assuming the amplitude of the AC driving voltage to be much smaller than the bias voltage, with the dimensionless quantities defined in (3), the nondimensional equation of motion is obtained:

$$\ddot{x} + \mu\dot{x} + \alpha x + \beta x^3 = \gamma\left(\frac{1}{(1-x)^2} - \frac{1}{(1+x)^2}\right) + \frac{A}{(1-x)^2}\mathrm{Sin}\omega\tau \tag{5}$$

where the new derivative operator, $(\dot{\cdot})$, denotes the derivative with respect to τ. It is worth mentioning that the corresponding potential can be described as below if the potential is set to be zero at $x = 0$:

$$V(x) = \frac{\alpha x^2}{2} + \frac{\beta x^4}{4} - \gamma\left(\frac{1}{1-x} + \frac{1}{1+x}\right) + 2\gamma \tag{6}$$

Fig. 2 shows that the number of equilibrium points changes when applied voltage varies. When the bias voltage does not exist there is only one stable state and the equilibrium point is a stable center point at $x = 0$. When the bias voltage exists; at a critical position the resonator becomes unstable and is deflected against one of the stationary transducer electrodes

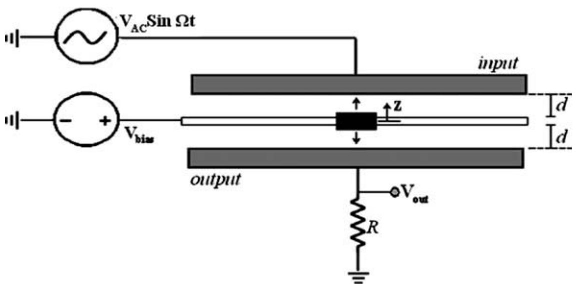

Fig. 1. A schematic picture of the electrostatically actuated micromechanical resonator.