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A circularly coupled oscillator system for relative phase regulation and its application to timing control of a multicylinder engine

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ABSTRACT

In this paper, distributed systems that consist of many locally connected subsystems, especially oscillators, and produce linear state relations, such as a state difference, are treated. The relations are defined between two connected subsystems, where their references are also assigned as a goal behavior simultaneously. The problem is: how subsystem dynamics are constructed to converge the relations to their references by only use of local operations, and how these references are adjusted if they are unachievable. To solve the above problems, a mathematical description of the subsystem interactions are clarified by extending a method based on the gradient dynamics. Then, the reference adjustment is defined so that the subsystem interactions decreases. As an example of this formulation, the relative phase control of the circularly coupled oscillator system is considered, where the oscillation with the uniform phase lag should be achieved. This oscillator system is applied to the timing controller for the multicylinder engine, and its effectiveness is discussed based on the simulation results.

1. Introduction

Spatiotemporal patterns are often observed in various fields [1]: convective rolls in a thermal liquid system [2], chemical oscillation in the BZ reaction [3], or neural activations or motions in biological systems [4–6]. These patterns are constructed from coordinated behavior of the many homogeneous components, such as molecules, neurons, or their ensembles. Although the effect of independent actions of the distributed elements is limited only to their neighborhood, such microscopic behavior produces a global pattern at the macroscopic level. This bottom–up approach allows the patterns to change with the situation. In the above examples, striped and hexagonal patterns emerge in convective patterns [2], or chemically impure substances determine whether a target or spiral pattern is produced [3]. The oscillatory behavior of a distributed system is occasionally described using a coupled oscillator system, where the pattern is represented by relative phases in the synchronization. Coupled oscillator systems are utilized to mathematically explain human motor behavior [7–9], quadruped locomotion [10,11], insects [12,13], swimming patterns [14,15], and even the actions of single cell amoeba [16]. They are effectively used in robots as a CPG (Central Pattern Generator) controller [17–19]. The mathematical analysis of coupled oscillator systems has been thoroughly discussed with numerical simulations [20–25]. This paper differs from these

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excellent studies in that the desired phase lags (i.e., reference relative phases) are initially explicitly assumed as the objective of the control and then the design of the oscillator interaction is considered to achieve this objective. Next, adaptive behavior is considered for adjusting the reference relative phases if they are inappropriate for the environment. This engineering viewpoint is import for providing more insight into the system design. As for the former problem, a method based on the gradient system was proposed [20]. In Section 2, this method is reviewed after formulating the problem with some assumptions, and then the mathematical formulation of the subsystem dynamics is discussed with focusing on the interactions. In Section 3, how is the situation in which the reference relations are unachievable is explained at first, and then an adjustment method of the unachievable reference is proposed based on the subsystem interactions. In Section 4, the relative phase regulation in coupled oscillator system is considered as an example, and is applied to the timing control of multicylinder engine. Finally, this paper is concluded in the Section 5.

2. Control in the distributed system

2.1. Problem setting

Yuasa and Ito proposed a method for controlling relative phases in a coupled oscillator system [20]. Their framework is based on parallel and distributed operations and is more general in the sense that linear relations between the one-dimensional state variables of two coupled subsystems can be regulated to their references. Of course, the linear relations include a difference in the state variables (the relative phase, in the case of a coupled oscillator system).

First, the following are assumed:

- The system under consideration consists of homogeneous subsystems.
- The state of the subsystem is represented by a scalar variable.
- The subsystems are connected locally, implying that there is no hub subsystem which all subsystems connect to.
- Two connected subsystems can exchange state variables, each affecting the state variable of the other. This is called "interaction".
- A constraint (i.e., a variable calculated from the states of the two coupled subsystems) is defined for each connection. This variable is called a "constraint variable".
- Each constraint variable possesses a reference value that represents a purpose of the entire system.

Under these assumptions, the problem is described as follows:

- Define the dynamics of each subsystem with local interaction such that the constraint variables are regulated to their references.

Here, dynamics with local interaction means, mathematically, that the system never contains any state variables other than those of the connected subsystems.

2.2. A design method based on a gradient system

The state of each subsystem is denoted by $q_i \in R(i = 1, \dots, M)$, where M is the number of the subsystems. Yuasa and Ito [20] formulate a case where the constraint variable p_k is given by the linear relation:

$$p_k = L_{ki}q_i - L_{kj}q_j \tag{1}$$

This equation implies that the connection $k(k = 1, \dots, K)$ connects subsystem i and subsystem j, as shown in Fig. 1. This relation can be represented using the matrix form

$$P = LQ \tag{2}$$

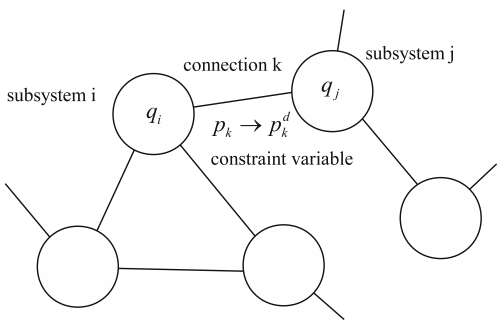

Fig. 1. System consisting of many homogeneous subsystem and constraint variable.