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Geometrically nonlinear bending analysis of laminated composite plate

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ABSTRACT

In this work, a transverse bending of shear deformable laminated composite plates in Green–Lagrange sense accounting for the transverse shear and large rotations are presented. Governing equations are developed in the framework of higher order shear deformation theory. All higher order terms arising from nonlinear strain–displacement relations are included in the formulation. The present plate theory satisfies zero transverse shear strains conditions at the top and bottom surfaces of the plate in von-Karman sense. A C^0 isoparametric finite element is developed for the present nonlinear model. Numerical results for the laminated composite plates of orthotropic materials with different system parameters and boundary conditions are found out. The results are also compared with those available in the literature. Some new results with different parameters are also presented.

1. Introduction

Composite laminates are extensively used for a number of applications in various engineering fields, where weight saving is crucial, mainly because of their high strength-to-weight, stiffness-to-weight ratios, good energy and sound absorption, and often also low production cost. They have also been widely used to build large proportions of aerospace, underwater and automotive structures. These structures represent a compromise between the stiffness and the lightness. Many global plate theories exist that account for transverse shear strains. Of these, the theories based on assumed displacement fields provide a background for the present theory. Based on the assumption of smallness of certain quantities of the formulation, the problem may be reduced to a linear problem. Linear solution may be obtained with considerable ease and less computational cost when compared to nonlinear solutions. In many instances, the assumption of linearity leads to a reasonable idealization of the behavior of the system. However, in some cases this may result in an unrealistic approximation of the response. The linear and nonlinear types of analysis depend on the goal of the analysis errors in the system response that may be tolerated. In some cases nonlinear analysis is the only option left for the analysis.

A considerable work has been done on the composites in the last three decades. For the commonly used composites, classical lamination theory (CLT) has been shown not to be capable of yielding accurate results due to severe shear deformation from the less stiff matrix. In order to take into account the larger shear deformation, many modified laminated plate theories were proposed by using revised displacement fields of different order [1–5]. Large deflection analysis of laminated composite plates in von-Karman sense has been attempted extensively in the literature [3–17]. Chang and Huang [3] presented a finite element analysis of composite laminates based on the higher order displacement theory allowing both the transverse shear and normal deformations using von-Karman strain–displacement relations. Reddy and Chao [4] employed the shear deform-

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able theory accounting for the transverse shear (in the sense of Reissner–Mindlin's thick plate theory) and large rotations (in the sense of von-Karman theory) in the construction of a variational statement and associated finite element models of layered composite plates. Reddy [5] developed a higher order shear deformation theory (HSDT) of plates accounting for the von-Karman strains and containing the same dependent unknowns as in the Hencky–Mindlin type first order shear deformation theory to predict the deflection. Chia [6] solved the nonlinear bending of an unsymmetrically laminated angle-ply rectangular plate analytically under lateral load satisfying the von-Karman-type strain. Putcha and Reddy [7] developed a refined mixed finite element model consists of 11 degrees of freedom (three displacements, two rotations, and six moment resultants) per node based on the HSDT. The element is numerically evaluated for accuracy in static analyses of layered anisotropic plates incorporating the von-Karman strains. Striz et al. [8] investigated the behavior of thin, circular, isotropic elastic plates with immovable edges and undergoing large deflections. They used the Newton–Raphson technique to solve the nonlinear systems of equations. Chandrasekharappa and Srirangarajan [9] studied the nonlinear static and dynamic analysis of clamped rectangular composite plate under step pressure pulse load excitation considering the material damping in the von-Karman relations. They solved the coupled nonlinear partial differential equations by a one-term solutions and applying Galerkin's technique to the deflection equation. Bert et al. [10] investigated the behavior of thin, rectangular, orthotropic elastic plates, with immovable edges and undergoing large deflections, by the numerical technique of differential quadrature. They obtained the approximate results using the Newton–Raphson method and a finite difference-based method to solve the nonlinear systems of equations. Singh et al. [11] investigated the large deflection bending analysis of antisymmetric rectangular cross-ply plates based on von-Karman plate theory, with one-term approximation for the in-plane and transverse displacements, under sinusoidal loading. Li et al. [12] developed incremental load technique for solving the bending problem of a thin circular plate with large deflection. Clarke et al. [13] described various incremental-iterative techniques based on the Newton Raphson approach to analyze the geometric nonlinear behavior. Urthaler and Reddy [14] presented the nonlinear bending analysis of laminated composite plates using mixed finite element model which include geometric nonlinearity in the sense of the von-Karman based on the FSDT. Reddy and Chandrashekhara [15] developed a dynamic, shear deformation theory for doubly curved shell using finite element. They have taken von-Karman strain–displacement relations for geometrically nonlinear transient analysis of laminated composite shells. Shukla and Nath [16] presented the analytical solutions to the geometrically nonlinear boundary value problems of laminated composite plate undergoing moderately large deformations and subjected to various boundary conditions. They have used the FSDT and von-Karman-type nonlinearity. Nath et al. [17] studied nonlinear static and dynamic analysis for composite laminated anti-symmetric square plates supported on nonlinear elastic foundation subjected to uniformly distributed transverse and step loading, respectively, using the Chebyshev series technique. Their formulation is based on FSDT and von-Karman nonlinearity. Civalek [18,19] introduced a coupled methodology for the numerical solution of geometrically nonlinear static and dynamic problem of thin rectangular plates resting on elastic foundation in von-Karman sense. Srinivas and Rao [20] developed a three-dimensional, linear, small deformation theory of elasticity solution for the bending, vibration and buckling of simply supported thick orthotropic rectangular plates and laminates. Singh and Rao [21] developed a rectangular quadratic finite element which includes large displacements, strains and rotations. It incorporated Green strain in the framework of first order shear deformation theory.

It is evident from the literatures that most of the studies use von-Karman type of strain–displacement relation for the nonlinear bending analysis. A few studies on nonlinear bending analysis use Green–Lagrange strains. But, the large deformation response of composite plate having full nonlinearity in Green–Lagrange sense based on the HSDT incorporating all higher order terms has received no attention to the best of authors' knowledge.

In the present study, the nonlinear bending analysis of the laminated composite plates is presented in the framework of the HSDT. The geometric nonlinearity is modeled in Green–Lagrange sense. As a priori the zero transverse shear stress conditions on the top and bottom of the plate are satisfied in the displacement field in von-Karman sense. The system equations are the first derived using variational approach. A C^0 isoparametric plate finite element model with 10 field variables is proposed. Nonlinear fundamental frequencies are obtained using direct iteration scheme. Numerical results are presented with different parameters.

2. Theory

2.1. Kinematics

The geometry and origin of the material coordinates are at the middle of the laminate as shown in Fig. 1. The displacement field for the laminate is assumed as,

$$\begin{aligned}\bar{u}(x,y,z) &= u(x,y) + z\psi_x(x,y) + z^2\xi_x(x,y) + z^3\zeta_x(x,y)\\ \bar{v}(x,y,z) &= v(x,y) + z\psi_y(x,y) + z^2\xi_y(x,y) + z^3\zeta_y(x,y)\\ \bar{w}(x,y,z) &= w(x,y) + z\psi_z(x,y)\end{aligned} \quad (2.1)$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ denote the displacements of a point along the (x,y,z) coordinates. u, v and w are corresponding displacements of a point on the mid plane. ψ_x and ψ_y are the rotations of normal to the mid plane about the y-axis and x-axis, respectively. The functions ς_x, ς_y, ξ_x, ξ_y and ψ_z are the higher-order terms in the Taylor series expansion defined in the mid-plane of the plate. These functions are determined in terms of primary field variables using the assumptions that the transverse