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Projective lag synchronization of spatiotemporal chaos via active sliding mode control

Yuan Chai [a], Li-Qun Chen [a,b,c,*]

[a] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, PR China
[b] Department of Mechanics, Shanghai University, Shanghai 200444, PR China
[c] Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, Shanghai 200072, PR China

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ABSTRACT

This paper investigates projective lag synchronization of spatiotemporal chaos with disturbances. A control scheme is designed via active sliding mode control. The synchronization of spatiotemporal chaos between a drive system and a response system with disturbances and time-delay is implemented by adding the active sliding mode controllers. The control law is applied to two identical spatiotemporal Gray–Scott systems. Numerical results demonstrate the feasibility and the effectiveness of the proposed approach.

1. Introduction

Owing to its great potential for applications, control and synchronization of chaos have aroused considerable interests in many areas since the pioneering works of Ott et al. [1] and Pecora and Carroll [2] in 1990. After that many approaches have been proposed for chaos synchronization, such as adaptive control [3], active control [4], backstepping control [5], sliding mode control [6–9], impulsive control [10], etc. Recently based on the advantages of the active control and the sliding mode control method, a new method called "active sliding mode control technique" is proposed to realize chaos synchronization, Haeri and Emadzadeh [11] and Haeria et al. [12] investigated the synchronizing different chaotic systems and uncertain chaotic systems using active sliding mode control. Zhang et al. [13] reported on the synchronization of chaotic systems with parametric uncertainty using active sliding mode control. Tavazoei and Haeri [14] discussed the determination of active sliding mode controller parameters in synchronizing different chaotic systems.

An interesting synchronization phenomenon has been created, called the modified projective synchronization [15], which the states of the drive and response systems synchronize up to a constant scaling matrix. The modified projective synchronization includes the complete synchronization, anti-synchronization and projective synchronization [16–19]. Therefore, modified projective synchronization is worth research because of its diversity. And some research results for the modified projective synchronization have been obtained in recent years [20,21]. It is well known that time delay is ubiquitous when signals are communicated among neurons or secure communications [22,23], it is reasonable to require one system to synchronize the other system at a constant time delay [24]. In the research area of lag synchronization, several results have been appeared in the literature [25–27]. Therefore, we propose projective lag synchronization, which the states of the drive and response systems lag synchronize up to a constant scaling matrix. The dimension of the constant scaling matrix and systems are the same. The complete synchronization, anti-synchronization and projective synchronization belong to the projective

* Corresponding author at: Department of Mechanics, Shanghai University, Shanghai 200444, PR China. Tel.: +86 21 66136905.
E-mail address: lqchen@straff.shu.edu.cn (L.-Q. Chen).

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lag synchronization, which may be an appropriate technique to clearly indicate the synchronization of nature and artificial systems.

Most of the above studies are realized without any external perturbations. However, the noise disturbance is inevitable from a practical point of view. The chaotic systems are always in a changing environment and disturbed by some unknown factors from environment [28]. A small perturbation to spatiotemporal chaos will result in a drastic change in the chaotic behavior of the systems. Synchronization of chaos is unavoidably subject to external disturbances. Therefore, investigation of synchronization for the chaotic systems by the impact of artificially adding disturbances has become an important research topic [29]. Cai et al. [30] considered the modified projective synchronization of chaotic systems with disturbances via active sliding mode control.

Temporal chaos systems are selected for demonstration in most of papers, though a large number of natural systems are of spatiotemporal chaos [31]. Spatiotemporal chaos synchronization has greater potential application value to many fields [32], such as secure communication, physics, auto-control, fluid, chemical, and biological systems. Chian et al. [33] discussed the amplitude-phase synchronization at the onset of permanent spatiotemporal chaos. Therefore, the synchronization of spatiotemporal chaos has values of definite magnitude. However, the projective lag synchronization of spatiotemporal chaos with disturbances is still less reported, the research in spatiotemporal chaos synchronization via active sliding mode control has a vast unknown space to explore.

Yielding fruitful results put an important step forward from the investigation of theory. Motivated by the above analysis, in this paper, we consider projective lag synchronization of identical spatiotemporal chaotic systems with disturbances. Based on the active sliding mode control technique, the sufficient conditions are given to assure the valid projective lag synchronization occurs. Numerical results demonstrate the feasibility and the effectiveness of the proposed approach.

2. Active sliding mode controller design

Consider a spatiotemporal chaotic system described by the following nonlinear differential equation:

$$\frac{\partial u(x,t)}{\partial t} = Au(x,t) + F(u(x,t)), \tag{1}$$

where $u(x,t) = [u_1(x,t), u_2(x,t), \ldots, u_n(x,t)]^T \in R^n$ denotes the system's n-dimensional state vector, $A \in R^{n\times n}$ represents the constant matrices of the linear part and $F: R^n \to R^n$ is the nonlinear part of the system. Eq. (1) represents the master system. The controller $U(x,t) \in R^n$ is added into the slave system, it is given by

$$\frac{\partial v(x,t)}{\partial t} = Av(x,t) + F(v(x,t)) + U(x,t), \tag{2}$$

where $v(x,t) = [v_1(x,t), v_2(x,t), \ldots, v_n(x,t)]^T \in R^n$ is the slave system's n-dimensional state vector.

We consider two identical spatiotemporal chaotic systems with disturbances, which are described as follows

$$\frac{\partial u(x,t)}{\partial t} = Au(x,t) + F(u(x,t)) + D_1(t), \tag{3}$$

$$\frac{\partial v(x,t)}{\partial t} = Av(x,t) + F(v(x,t)) + D_2(t) + U(x,t), \tag{4}$$

where $D_1(t)$, $D_2(t)$ are the disturbances.

If there exists a constant matrix $P = \mathrm{diag}\{P_1, P_2, \ldots, P_n\}$ and P denotes a "scaling matrix". such that $\lim_{t\to\infty}\|u(x,t-\tau) - Pv(x,t)\| = 0$, then we call the systems (1) and (2) achieve the "projective lag synchronization", $\|D_1(t-\tau)\| < \delta_1$, $\|PD_2(t)\| < \delta_2$, where δ_1, δ_2 are known constants. Owing to properties of bounded disturbances, δ_1 and δ_2 are the maximum values of the disturbances $\|D_1(t-\tau\|)$ and $\|PD_2(t)\|(\|D_1(t-\tau)\| \leqslant \delta_1$ and $\|PD_2(t)\| \leqslant \delta_2)$. The practical effect of two values is an upper limit to $\|D_1(t-\tau)\|$, $\|PD_2(t)\|$, respectively.

Projective lag synchronization is the aggregation of complete synchronization, anti-synchronization and projective synchronization. All the dynamical states of projective lag synchronization are amplified or reduced synchronously. However, projective lag synchronization allows us to flex the scales of the different states independently because there are many scaling factors in a scaling matrix P.

Define the error state as $e(x,t) = u(x,t-\tau) - Pv(x,t)$, then the dynamics of the synchronization error can be expressed as

$$\begin{aligned}\frac{\partial e(x,t)}{\partial t} &= \frac{\partial u(x,t-\tau)}{\partial t} - P\frac{\partial v(x,t)}{\partial t} = Au(x,t-\tau) + F(u(x,t-\tau)) + D_1(t-\tau) - P(Av(x,t) + F(v(x,t)) + D_2(t) + U(x,t)) \\ &= Ae(x,t) + F(u(x,t-\tau)) - PF(v(x,t)) + D_1(t-\tau) - PD_2(t) - PU(x,t) = Ae(x,t) + F(x,t) + D(t) - PU(x,t),\end{aligned} \tag{5}$$

where $F(x,t) = F(u(x,t-\tau)) - PF(v(x,t))$ and $D(t) = D_1(t-\tau) - PD_2(t)$.

To synchronize the systems is to find a controller, such that the error system (5) is asymptotically stable, which implies the projective lag synchronization between the systems (3) and (4) is realized, i.e.

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|u(x,t-\tau) - Pv(x,t)\| = 0. \tag{6}$$