Commun Nonlinear Sci Numer Simulat 14 (2009) 3498–3502

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Short communication

Maximum-revenue tariff under Bertrand duopoly with unknown costs

Fernanda A. Ferreira *, Flávio Ferreira

ESEIG, Instituto Politécnico do Porto, R.D. Sancho I, 981, 4480-876 Vila do Conde, Portugal

A R T I C L E I N F O

Article history:
Received 21 November 2008
Accepted 28 January 2009
Available online 5 February 2009

PACS:
89.65.Gh

Keywords:
Game theory
Industrial organization
Optimization
Bertrand model
Tariffs
Uncertainty

A B S T R A C T

This paper considers an international trade under Bertrand model with differentiated products and with unknown production costs. The home government imposes a specific import tariff per unit of imports from the foreign firm. We prove that this tariff is decreasing in the expected production costs of the foreign firm and increasing in the production costs of the home firm. Furthermore, it is increasing in the degree of product substitutability. We also show that an increase in the tariff results in both firms increasing their prices, an increase in both expected sales and expected profits for the home firm, and a decrease in both expected sales and expected profits for the foreign firm.

1. Introduction

Tariff revenue may be an important source of government revenue for developing countries that do not have an efficient tax system. So, the government may use the maximum-revenue tariff. Brander and Spencer [2] have shown that a tariff has a profit-shifting effect in addition to its effect on tariff revenue. Larue and Gervais [8] studied the effect of maximum-revenue tariff in a Cournot duopoly. The propose of this paper is to examine the maximum-revenue tariff under international Bertrand competition with differentiated products when rivals' production costs are unknown. Clarke and Collie [3] studied a similar question, when there is no uncertainty on the production costs. Bertrand duopoly when rivals' production costs are unknown was analyzed by Ferreira and Pinto [6,7] and by Spulber [9], but in a model without tariffs. The effects of production costs uncertainty were also analyzed by Ferreira et al. [4,5] in other duopoly models. In this paper, we consider a two-country, two-good model where a domestic and a foreign good are produced by a home and a foreign monopolist, respectively. Since we assume that the two countries are perfectly symmetric, it is sufficient to describe only the domestic economy. We suppose that each firm has two different technologies, and uses one of them according to a certain probability distribution. The use of either one or the other technology affects the unitary production cost. Both probability distributions of unitary production costs are common knowledge. We do ex-ante and ex-post analyses. We prove that the tariff imposed by the home government is decreasing in the expected production costs of the foreign firm and increasing in the production costs of the home firm. Furthermore, it is increasing in the degree of product substitutability. We also show that an increase in the tariff results in both firms increasing their prices, an increase in both expected sales and expected profits for the home firm, and a decrease in both expected sales and expected profits for the foreign firm.

* Corresponding author. Tel.: +351252291700; fax: +351252291714.
E-mail addresses: fernandaamelia@eu.ipp.pt (F.A. Ferreira), flavioferreira@eu.ipp.pt (F. Ferreira).

1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2009.01.026

2. The model and the equilibrium

There are two countries, home and foreign. Each country has one firm, firm F_1 (home firm) and firm F_2 (foreign firm), that produces differentiated goods. Consider the home market, where the two firms compete in a Bertrand duopoly, i.e., the firms simultaneously choose prices, respectively, $p_1 \geqslant 0$ and $p_2 \geqslant 0$ (see [1] or, for example [10]). The direct demands are given by

$$q_i = a - p_i + bp_j,$$

where $i, j \in \{1, 2\}$ with $i \neq j$, q_i stands for quantity, $a > 0$ is the intercept demand parameter and $b \geqslant 0$ is a constant representing how much the product of one firm is a substitute for the product of the other (see, for example [10]). For simplicity, we assume $b \leqslant 1$. These demand functions are unrealistic in that one firm could conceivably charge an arbitrary high price and still have a positive demand provided the other firm also charges a high enough price. However, this function is chosen to represent a linear approximation to the "true" demand function, appropriate near the usual price settings where the equilibrium is reached. Each firm has two different technologies, and uses one of them following a certain probability distribution. The use of either one or the other technology affects the unitary production cost. The following probability distributions of the firms' production costs are common knowledge among both firms:

$$C_1 = \begin{cases} c_A & \text{with probability } \phi, \\ c_B & \text{with probability } 1 - \phi, \end{cases}$$
$$C_2 = \begin{cases} c_H & \text{with probability } \theta, \\ c_L & \text{with probability } 1 - \theta. \end{cases}$$

We suppose that $c_A > c_B$, $c_H > c_L$ and $c_A, c_B, c_H, c_L < a$. Moreover, we suppose that the highest unitary production cost of any firm is greater than the lowest unitary production cost of the other one, that is, $c_A > c_L$ and $c_H > c_B$. The government in the home country imposes a specific import tariff of t per unit of imports from the foreign firm. Firms' profits, π_1 and π_2, are given by

$$\pi_1(p_1(c_1), p_2(c_2)) = (a - p_1(c_1) + bp_2(c_2))(p_1(c_1) - c_1),$$
$$\pi_2(p_1(c_1), p_2(c_2)) = (a - p_2(c_2) + bp_1(c_1))(p_2(c_2) - c_2 - t),$$

where the price $p_i(c_i)$ depends on the unitary production cost c_i of firm F_i, for $i \in \{1, 2\}$.

Lemma 1. *Let $E(C_1) = \phi c_A + (1 - \phi)c_B$ be the expected unitary production cost of firm F_1, and let $E(C_2) = \theta c_H + (1 - \theta)c_L$ be the expected unitary production cost of firm F_2. Assuming an interior solution where both firms sell positive quantities in the home country market, the Bertrand equilibrium prices are given by*

$$p_1^*(c_A) = \frac{2a(2 + b) + (4 - b^2)c_A + b^2E(C_1) + 2b(E(C_2) + t)}{2(4 - b^2)}, \tag{1}$$

$$p_1^*(c_B) = \frac{2a(2 + b) + (4 - b^2)c_B + b^2E(C_1) + 2b(E(C_2) + t)}{2(4 - b^2)}, \tag{2}$$

$$p_2^*(c_H) = \frac{2a(2 + b) + (4 - b^2)c_H + 4t + b^2E(C_2) + 2bE(C_1)}{2(4 - b^2)}, \tag{3}$$

$$p_2^*(c_L) = \frac{2a(2 + b) + (4 - b^2)c_L + 4t + b^2E(C_2) + 2bE(C_1)}{2(4 - b^2)}. \tag{4}$$

Proof. If firm F_1's unitary production cost is high, $p_1^*(c_A)$ is the solution of

$$\max_{p_1 \geqslant 0}(\theta(a - p_1 + bp_2^*(c_H))(p_1 - c_A) + (1 - \theta)(a - p_1 + bp_2^*(c_L))(p_1 - c_A));$$

and if it is low, $p_1^*(c_B)$ is the solution of

$$\max_{p_1 \geqslant 0}(\theta(a - p_1 + bp_2^*(c_H))(p_1 - c_B) + (1 - \theta)(a - p_1 + bp_2^*(c_L))(p_1 - c_B)).$$

If firm F_2's unitary production cost is high, $p_2^*(c_H)$ is the solution of

$$\max_{p_2 \geqslant 0}(\phi(a - p_2 + bp_1^*(c_A))(p_2 - c_H - t) + (1 - \phi)(a - p_2 + bp_1^*(c_B))(p_2 - c_H - t));$$

and if it is low, $p_2^*(c_L)$ is the solution of

$$\max_{p_2 \geqslant 0}(\phi(a - p_2 + bp_1^*(c_A))(p_2 - c_L - t) + (1 - \phi)(a - p_2 + bp_1^*(c_B))(p_2 - c_L - t)).$$

Solving these maximization problems, we obtain equalities (1)–(4) . □