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Almost periodic solutions for Lotka–Volterra systems with delays ☆

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ABSTRACT

This paper studies a general class of delayed almost periodic Lotka–Volterra system with time-varying delays and distributed delays. By using the definition of almost periodic function, the sufficient conditions for the existence and uniqueness of globally exponentially stable almost periodic solution are obtained. The conditions can be easily reduced to special cases of cooperative systems and competitive systems.

1. Introduction

Lotka–Volterra system is one of the most celebrated models in mathematical biology and population dynamics. In recent years, it has also been found with successful and interesting applications in physics, chemistry, economics and other areas (see [1–6,9]). Moreover, in [7], it was shown that the continuous-time recurrent neural networks can be embedded into Lotka–Volterra models by changing coordinates, which suggests that the existing techniques in the analysis of Lotka–Volterra systems can also be applied to recurrent neural networks.

Due to its theoretical and practical significance, the Lotka–Volterra system has been extensively and intensively studied (see [8–15] and the cites therein). Since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in a fluctuating environment. Therefore, on the one hand, models should take into account both the seasonality of the periodically changing environment and the effects of time delays. However, on the other hand, in fact, it is more realistic to consider almost periodic system than periodic system.

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For the periodic Lotka–Volterra systems, many skills and techniques have been developed. The existence of positive periodic solutions for such systems can be obtained by fixed point theorem [9], by method of Lyapunov functions [8], by the theory of monotone semiflows generated by functional differential equations [11], or by the Mawhin's continuation theorem of coincidence degree principle [12]. Comparably, there are few methods to analyze the almost periodic systems. Most of the papers on almost periodic system are based on the constructed appropriate Lyapunov functions [15], or almost periodic functional hull theory [14].

Motivated by paper [9], in this paper, we directly analyze the right function of almost periodic system, and obtain the sufficient conditions for the existence and uniqueness of globally exponentially stable almost periodic solution by the definition of almost periodic function. The model considered here is a very general form of Lotka–Volterra systems, including cooperative systems, competitive systems and their hybrids. Conclusions can be easily reduced to special cases of cooperative systems and competitive systems. On the other hand, we note that suppose $b_i(t) \geqslant 0$ in Teng [13] and Meng [14], then we can extend it to the case that $\lim_{T\to+\infty} \frac{1}{T}\int_0^T b_i(u)du > 0$.

The organization of this paper is as follows. In the following section, model description and some preliminaries are given. In Section 3, a sufficient condition for globally exponentially stable almost periodic solution is obtained and a numerical example is given at last.

2. Preliminaries

Consider the following general nonautonomous Lotka–Volterra type multispecies systems with time-varying delays and distributed delays:

$$\frac{dx_i(t)}{dt} = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) - \sum_{j=1}^{n}\int_{-\sigma_{ij}}^{0} x_j(t+s)d_s\mu_{ij}(t,s)\right], \quad i = 1,2,\dots,n, \tag{1}$$

with the initial condition

$$x_i(s) = \phi_i(s) \quad \text{for } s \in [-\tau, 0], \tag{2}$$

where $\tau = \max\{\sigma_{ij}, i,j = 1,\dots,n\}$, $\phi_i(s)$ is bounded continuous functions on $[-\tau,0]$ and $\phi_i(s) > 0$, for all $i = 1,2,\dots,n$. Throughout this paper, we always assume that the following assumptions hold for system (1):

(H1) Functions $b_i(t)$, $a_{ij}(t)$ $(i,j = 1,2,\dots,n)$ are continuous and almost periodic such that

$$\lim_{T\to+\infty}\frac{1}{T}\int_0^T b_i(u)du > 0, \quad a_{ii}(t) > 0.$$

(H2) Functions $\mu_{ij}(t,s)$ are continuous for any $s \in \mathbb{R}$ and for any fixed $t \in \mathbb{R}$, $d_s\mu_{ij}(t,s)$ are Lebesgue–Stieljies measures for all $i,j = 1,2,\dots,n$.

(H3) For any $\varepsilon > 0$, there is a constant $l = l(\varepsilon) > 0$, such that in any interval $[\alpha, \alpha + l]$ there exists ω such that these inequalities

$$|b_i(t+\omega) - b(t)| < \varepsilon, \quad |a_{ij}(t+\omega) - a_{ij}(t)| < \varepsilon, \quad \int_{-\sigma_{ij}}^{0} |d_s\mu_{ij}(t+\omega,s) - d_s\mu_{ij}(t,s)| < \varepsilon,$$

are satisfied for all $t \in \mathbb{R}$, where $i,j = 1,2,\dots,n$.

We further use the following definitions.

Defintion 1. Suppose $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ is any one solution for system (1) with the initial condition (2), $x(t)$ is said to be a positive solution in $\mathbb{R}^n$, if for $t \in \mathbb{R}$ and $i = 1,2,\dots,n$ such that $x_i(t) > 0$.

Defintion 2. The $\{\xi,\infty\}$-norm of a vector $x \in \mathbb{R}^n$ is defined by

$$\|x(t)\|_{\{\xi,\infty\}} = \max_{i=1,2,\dots,n} |\xi_i^{-1}x_i(t)|,$$

where $\xi = (\xi_1, \xi_2, \dots, \xi_n)$, $\xi_i > 0$ for all $i = 1,2,\dots,n$.

Defintion 3. An almost periodic solution $y(t)$ of system (1) is said to be globally exponentially stable with convergence rate $\beta > 0$, if for any positive solution $x(t)$ of system (1) such that

$$\|x(t) - y(t)\|_{\{\xi,\infty\}} = O(e^{-\beta t}), \quad t \to \infty.$$

Defintion 4. The ith and the jth species are cooperative or competition if $a_{ij}(t)$ and $a_{ji}(t)$ $(i \neq j)$ are both negative or both positive, respectively.

Defintion 5. Model (1) is called a cooperative system if $a_{ij}(t) \leqslant 0$ for all $i \neq j$, and $d_s\mu_{ij}(t,s) \leqslant 0$ for all $i,j = 1,2,\dots,n$. On the other hand, model (1) is called a competitive system if $a_{ij}(t) \geqslant 0$ for all $i \neq j$, and $d_s\mu_{ij}(t,s) \geqslant 0$ for all $i,j = 1,2,\dots,n$.

Moreover, the following lemma guarantees the positivity of the solution of system with the initial condition (2).