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Study of hydromagnetic heat and mass transfer flow over an inclined heated surface with variable viscosity and electric conductivity

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ABSTRACT

The effects of variable electric conductivity and temperature dependent viscosity on hydromagnetic heat and mass transfer flow along a radiate isothermal inclined permeable surface in a stationary fluid in the presence of internal heat generation (or absorption) are analyzed numerically presenting local similarity solutions for various values of the physical parameters. The research shows that the difference in the results between variable Prandtl number and constant Prandtl number are significant when fluid viscosity strongly dependents on the temperature. The results also show that skin friction coefficient, Nusselt number and Sherwood number are lower for the fluids of constant electric conductivity than those of the variable electric conductivity.

1. Introduction

The study of thermal boundary layer flows of variable viscosity on isothermal heated surfaces not only possesses theoretical interest but also models many fluid transport mechanisms encountered in industries and engineering systems. Amongst others, we can name hot rolling, wire drawing, glass fiber production, paper production, gluing of labels on hot bodies, drawing of plastic films, etc. When a cooler fluid flows around a hot body, the temperature of the fluid will rise in a thin layer around the body and in a wake behind it. This thin layer is known as the thermal boundary layer. In this layer, flow and thermal phenomena interact nonlinearly and governed by the so-called thermal boundary layer equations. In classical treatment of thermal boundary layers, the kinematic viscosity is assumed to be constant; however, experiments indicate that this assumption only makes sense if temperature does not change rapidly for the application of interest. Indeed, for liquids, experimental data shows that viscosity decreases with temperature.

Viscosity changes with temperature, for example the absolute viscosity of water decreases by 240% when the temperature increases from 10 °C to 50 °C which has been shown by Herwig and Wickern [1]. Film of fluids with constant viscosity along an inclined heated plate was investigated by Saouli and Saouli [2]. Meanwhile, several authors have investigated the effects of temperature dependent viscosity on the flow of non-Newtonian fluids in a channel under various conditions (e.g. Makinde [3], Szeri and Rajagopal [4], Yurusoy and Pakdemirli [5]). Ali [6] has studied the effect of

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Nomenclature

B_0	applied magnetic field ($\mathrm{Wb\,m^{-2}}$)
C	species concentration ($\mathrm{kg\,m^{-2}}$)
Cf	skin friction coefficient
C_p	specific heat at constant pressure ($\mathrm{J\,kg^{-1}\,K^{-1}}$)
C_w	concentration at the porous plate ($\mathrm{kg\,m^{-2}}$)
C_∞	species concentration at infinity ($\mathrm{kg\,m^{-2}}$)
D_m	molecular diffusivity ($\mathrm{m^{-2}\,s^{-1}}$)
f	dimensionless stream function
f_W	dimensionless suction parameter
g	acceleration due to gravity ($\mathrm{m\,s^{-2}}$)
Gr	Grashof number
Gc	modified Grashof number
Nu_x	local Nusselt number
Pn	radiative Prandtl number
Pr	variable Prandtl number
Pr_∞	ambient Prandtl number
q_w	surface heat flux ($\mathrm{W\,m^{-2}}$)
Q	local heat generation parameter
Q_0	heat generation parameter (W)
R	radiation parameter
Re_x	local Reynolds number
Sc	Schmidt number
Sh_x	local Sherwood number
T	temperature within boundary layer (K)
T_w	temperature at the plate (K)
T_∞	temperature of the ambient fluid (K)
u	velocity along x-axis ($\mathrm{m\ s^{-1}}$)
U_0	characteristic velocity ($\mathrm{m\ s^{-1}}$)
U_∞	velocity outside the boundary layer ($\mathrm{m\ s^{-1}}$)
v	velocity along y-axis ($\mathrm{m\ s^{-1}}$)
v_0	suction velocity ($\mathrm{m\ s^{-1}}$)
x	coordinate along the surface (m)
y	coordinate normal to the surface (m)

Greek symbols

α	angle of inclination (rad)
β	coefficient of volume expansion ($\mathrm{K^{-1}}$)
β^*	coefficient of volume expansion with concentration ($\mathrm{K^{-1}}$)
ρ_∞	mass density of the ambient fluid ($\mathrm{kg\,m^{-3}}$)
μ	coefficient of dynamic viscosity (Pa s)
υ	apparent kinematic viscosity ($\mathrm{m^{-2}\,s^{-1}}$)
σ	electric conductivity ($\mathrm{m\Omega\,m^{-1}}$)
σ_0	magnetic permeability ($\mathrm{N\,A^{-2}}$)
κ	thermal conductivity ($\mathrm{W\,m^{-1}K^{-1}}$)
η	similarity parameter
ψ	stream function ($\mathrm{m^{-2}\,s^{-1}}$)
θ	dimensionless temperature
Θ	viscosity parameter
ϕ	dimensionless concentration

temperature dependent viscosity on laminar mixed convection boundary layer flow and heat transfer on a continuously moving vertical surface. Laminar falling liquid film with variable viscosity along an inclined heated plate has been studied by Makinde [7]. A steady two-dimensional flow of an electrically conducting incompressible fluid over a heated stretching sheet with variable viscosity has been investigated by Mukhopadhyay et al. [8]. Pop et al. [9], and Elbashbeshy and Bazid [10] have studied the effect of variable viscosity using the similarity solution with no buoyancy force. Although viscosity varies with temperature, all of the afore-mentioned works considered constant Prandtl number (a parameter directly proportional to fluid viscosity, see Section 3) within the boundary layer. So one of the objectives of this study is to investigate the roll of variable Prandtl number on the heat and mass transfer flows.