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# Exact coherent structures for coupled integrable dispersionless equations

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ABSTRACT

In this paper, the singular manifold method is applied to search coherent structures in an analytical form for the coupled integrable dispersionless equations. The Generalized solutions have been derived to the coupled integrable dispersionless equations, where the solutions are determined by the singular variable totally. With the aid of symbolic computation and plot representation of Maple, some coherent structures expressed in terms of new forms, such as solitoffs and breather lattice structures, have been illustrated by means of arbitrary functions in the analytical forms.



## 1. Introduction

For the following coupled integrable dispersionless equations [1]

$$u_{xt} + (vw)_x = 0, \tag{1a}$$
$$v_{xt} - 2vu_x = 0, \tag{1b}$$
$$w_{xt} - 2wu_x = 0. \tag{1c}$$

Konno and co-workers [1] have shown that the coupled system is solvable by using the inverse scattering method. Similar property was found by Alagesan and co-workers [2] when they investigated the singularity structure analysis of these coupled system and found that these system possesses the Painlevé property by the method of singular manifold analysis [3].

Recently, Naranmandula et al. [4], Dai et al. [5], and Liu et al. [6] have obtained some exact solutions and constructed some coherent structures by the method of improved homogeneous balance, exp-function and Riccati equation mixed method and Jacobi elliptic function expansion method, respectively. The existence of these different coherent structures also tells us that there are still more new coherent structures in the (1 + 1)-dimensional systems can be found, since many methods have been proposed and widely applied to solve (1 + 1)-dimensional nonlinear wave equations extensively [7–16], and these methods can also be applied to find more coherent structures in the (1 + 1)-dimensional systems. For example, the singular manifold analysis [3] has been used widely to analyze the integrability of nonlinear systems, (1 + 1)-dimensional, (2 + 1)-dimensional or higher dimensional. In fact, this method has been extended by Peng and his co-workers [17–20] to construct the localized solutions, such as dromions [21,22] and solitoffs [23] in the (2 + 1) or higher dimensional nonlinear systems and it is shown that the singular manifold method is powerful in this direction. In this paper, we will take the coupled integrable dispersionless Eqs. (1) as an example to show there are more coherent structures in the (1 + 1)-dimensional systems by applying the singular manifold method [3] in details.

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## 2. The analytical solutions to the coupled integrable dispersionless equations

According to the singular manifold method [3], the solutions to the coupled integrable dispersionless Eqs. (1) can be truncated as

$$u = \phi^{-1}u_0 + u_1, \tag{2a}$$

$$v = \phi^{-1}v_0 + v_1, \tag{2b}$$

$$w = \phi^{-1}w_0 + w_1, \tag{2c}$$

where $\phi = \phi(x,t)$ is the singular manifold variable, $u_i = u_i(x,t)$, $v_i = v_i(x,t)$, and $w_i = w_i(x,t)$, $i = 0,1$.

Substituting Eq. (2) into Eq. (1) and equating the coefficients with the same powers of $\phi$, one gets

$$u_0 = -\phi_t, \tag{3a}$$

$$v_0 w_0 = -\phi_t^2 \tag{3b}$$

and $u_1$, $v_1$ and $w_1$ satisfy the Eq. (1), similar results have been found by Alagesan and Porsezian [2].

Since many studies [4,6] have found that $v$ is proportional to $w$, here we take

$$v_0 = a\phi_t, \quad w_0 = -\frac{1}{a}\phi_t, \tag{4}$$

where $a$ is a non-zero constant.

Substituting (3) and (4) back into other equations for coefficients with the same powers of $\phi$, one gets

$$u_1 = \frac{\phi_{tt}}{2\phi_t} + h(t), \tag{5a}$$

$$v_1 = -\frac{a\phi_{tt}}{2\phi_t}, \tag{5b}$$

$$w_1 = \frac{\phi_{tt}}{2a\phi_t}, \tag{5c}$$

where $h(t)$ is an arbitrary function and $\phi$ satisfies the following equation

$$\left(\frac{\phi_{tt}}{\phi_t}\right)_{xt} - \left(\frac{\phi_{tt}}{\phi_t}\right)\left(\frac{\phi_{tt}}{\phi_t}\right)_x = 0, \tag{6}$$

i.e.

$$\left(\frac{\phi_{tt}}{\phi_t}\right)_t = \frac{1}{2}\left(\frac{\phi_{tt}}{\phi_t}\right)^2 + g(t), \tag{7}$$

where $g(t)$ is another arbitrary function.

If $\phi$ satisfies (6) or (7), then the solution to Eq. (1) can be written as

$$u = \left(-\frac{\phi_t}{\phi} + \frac{\phi_{tt}}{2\phi_t}\right) + h(t), \tag{8a}$$

$$v = -a\left(-\frac{\phi_t}{\phi} + \frac{\phi_{tt}}{2\phi_t}\right), \tag{8b}$$

$$w = \frac{1}{a}\left(-\frac{\phi_t}{\phi} + \frac{\phi_{tt}}{2\phi_t}\right). \tag{8c}$$

As mentioned in Ref. [3], if the arbitrary function $\phi$ takes a separable form, then (6) can be solved. For any given $\phi$, we can derive (8), generalized analytical solutions for Eq. (1). For example, if $\phi$ takes the following separable form

$$\phi = R(t) + f(x), \tag{9}$$

where $R(t)$ and $f(x)$ are two arbitrary functions. It is easy to check that $\phi$ satisfies (6), and then the solution to Eq. (1) can be written as

$$u = -\frac{R_t}{R(t) + f(x)} + \frac{R_{tt}}{2R_t} + h(t), \tag{10a}$$

$$v = -a\left[-\frac{R_t}{R(t) + f(x)} + \frac{R_{tt}}{2R_t}\right], \tag{10b}$$

$$w = \frac{1}{a}\left[-\frac{R_t}{R(t) + f(x)} + \frac{R_{tt}}{2R_t}\right]. \tag{10c}$$