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Study of magnetohydrodynamic pulsatile flow in a constricted channel

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ABSTRACT

The present work reports the study of steady and pulsatile flows of an electrically conducting fluid in a differently shaped locally constricted channel in presence of an external transverse uniform magnetic field. The governing nonlinear magnetohydrodynamic equations simplified for low conducting fluids are solved numerically by finite difference method using stream function-vorticity formulation. The analysis reveals that the flow separation region is diminished with increasing values of magnetic parameter. It is noticed that the increase in the magnetic field strength results in the progressive flattening of axial velocity. The variations of wall shear stress with increasing values of the magnetic parameter are shown for both steady and pulsatile flow conditions. The streamline and vorticity distributions in magnetohydrodynamic flow are also shown graphically and discussed.

1. Introduction

The phenomenon of fluid flow separation in pulsatile flow has received considerable attention during the last few decades due to its practical applications in engineering and bio-mechanical systems. One of the major tasks in analysing such flow situations is to control the boundary layer separation as it imposes certain limitations on design and operation of many mechanical devices. The industrial and biological fluids are significantly affected by the application of the external magnetic field (Barnothy [1]). Human blood constitutes a suspension of red cells containing hemoglobin (in the form of iron oxide) in plasma. It can be considered as an electrically low conducting fluid and are significantly affected by the application of external magnetic field (Vardanyan [2]). Blood flow in arterises is pulsatile in nature. The study of pulsatile blood flow in large arteries has been motivated by the researchers because vascular fluid dynamics and hemodynamic factors play an important role in the development and progression of atherosclerotic diseases (Glagov et al. [3], Ku [4] and Mittal et al. [5]).

Several researchers have analysed the steady laminar flow through a constricted tube or channel (Lee [6], Huwang and Seymour [7] and Mahapatra et al. [8]). The dynamics of pulsatile laminar flow phenomenon over a contraction or expansion has been investigated and reported by many researchers namely, Tutty [9], Liu and Yamaguchi [10], Mittal et al. [5], Bandyopadhyay and Layek [11] and references therein. The steady MHD flow of an electrically conducting fluid in a channel with irregular geometries have been studied by Pal et al. [12] and Midya et al. [13] under laminar flow conditions in presence of a uniform transverse magnetic field.

The main objectives of the present work are to analyse the flow behaviour and corresponding flow separation in steady and pulsatile flows of an electrically conducting fluid with low conductivity through a constricted channel in presence of an external transverse uniform magnetic field. In this analysis we have neglected the induced magnetic field since the magnetic Reynolds number (Re_m) is very small and is found to be valid for the flow of liquid metals, human blood etc. (Shercliff [14]). The unsteady governing equations of motions are solved numerically by finite difference method using the stream

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function-vorticity formulation. The effects of various flow parameters such as flow Reynolds number, magnetic parameter, Strouhal number etc. on the axial velocity, wall shear stress and streamline and vorticity distributions are analysed and discussed in details.

2. Governing equations

Consider a two-dimensional laminar flow of an electrically conducting fluid in a rectangular channel with constrictions on both the walls, separated by a distance L in the unconstricted region, in presence of a uniform magnetic field $\mathbf{B}$ perpendicular to the channel walls. The walls are assumed to be rigid and impermeable. We employ a Cartesian co-ordinate system $(\tilde{x}, \tilde{y})$ in which $\tilde{x}$-axis is along the lower wall in the direction of the flow and $\tilde{y}$-axis perpendicular to the walls, the origin being taken at the centre of the constriction in the lower wall (Fig. 1). With the assumption that the magnetic Reynolds number $Re_m(=UL\sigma\mu_m)$ for the flow is very small i.e., $Re_m \ll 1$, the unsteady two-dimensional equations of motion in MHD flow of an incompressible Newtonian fluid, taking into account the Lorentz force, are written as

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}} + v\nabla^2\tilde{u} + \frac{1}{\rho}(\mathbf{J}\times\mathbf{B})_x \tag{1}$$

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{y}} + v\nabla^2\tilde{v} \tag{2}$$

and the equation of continuity is

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0. \tag{3}$$

Here $\tilde{p}$ is the pressure, ρ the density, v the kinematic viscosity, U the characteristic flow velocity (equal to the average velocity in a period over the inlet section). $\tilde{u}$ and $\tilde{v}$ are the component of velocity components along the $\tilde{x}$-axis and $\tilde{y}$-axis, respectively. $\mathbf{J} \equiv (J_x, J_y, J_z)$ is the current density, $\mathbf{B} \equiv (B_x, B_y, B_z) = (0, B_0, 0)$ the magnetic field, σ the electrical conductivity, μ_m the magnetic permeability of the medium.

Since the electric current flows along the normal to the plane of flow, we have from Ohm's law

$$J_x = 0, \quad J_y = 0, \quad J_z = \sigma[E_z + \tilde{u}B_0] \tag{4}$$

where E_z is the electric field along the z-direction (normal to the plane of the flow) and B_0 is the uniform magnetic field strength. Again for steady flow, Maxwell's equation $\nabla \times \mathbf{E} = 0$ gives $E_z = constant$, where $\mathbf{E}$ is the electric field acting along z-axis. In the present analysis, we have assumed that $E_z = 0$.

In view of the above assumptions the Eqs. (1) and (2) reduce to the following forms

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{x}} + v\nabla^2\tilde{u} - \frac{\sigma B_0^2\tilde{u}}{\rho} \tag{5}$$

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{p}}{\partial \tilde{y}} + v\nabla^2\tilde{v} \tag{6}$$

Introducing the following dimensionless quantities as

$$x = \frac{\tilde{x}}{L}, \quad y = \frac{\tilde{y}}{L}, \quad u = \frac{\tilde{u}}{U}, \quad v = \frac{\tilde{v}}{U}, \quad t = \frac{\tilde{t}}{T}, \quad p = \frac{\tilde{p}}{\rho U^2}, \quad Re = \frac{UL}{v}, \quad St = \frac{L}{UT}, \quad M = B_0 L\sqrt{\frac{\sigma}{\rho v}} \tag{7}$$

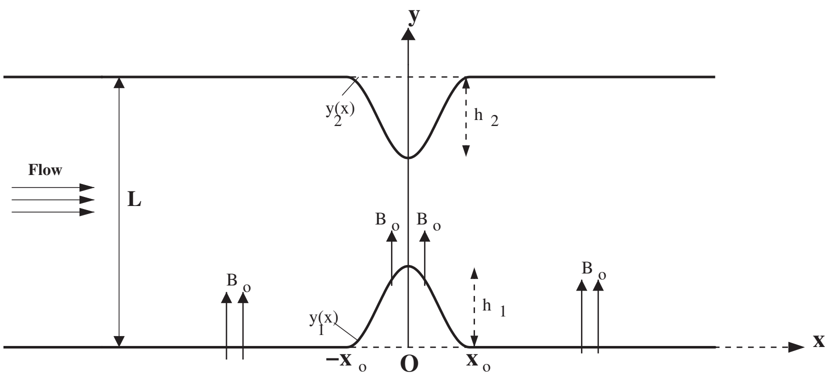

Fig. 1. Schematic diagram of the physical problem.