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Mixed convection flow of couple stress fluid between parallel vertical plates with Hall and Ion-slip effects

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ABSTRACT

An analysis is presented to investigate the Hall and Ion-slip effects on fully developed electrically conducting couple stress fluid flow between vertical parallel plates in the presence of a temperature dependent heat source. The governing non-linear partial differential equations are transformed into a system of ordinary differential equations using similarity transformations and then solved using homotopy analysis method (HAM). The effects of the magnetic parameter, Hall parameter, Ion-slip parameter and couple stress fluid parameter on velocity and temperature are discussed and shown graphically.

1. Introduction

Heat transfer in free and mixed convection in vertical channel has been the focus of extensive investigation for many decades due to its wide range of applications in the design of cooling systems for electronic devices and in the field of solar energy collection, etc. Heat exchanger technology involves convective flows in vertical channels. Several researchers have studied analytically and mostly numerically the problem of mixed convection heat transfer and fluid flow between vertical parallel plates. Aung and Worku [1] presented an exact solution for fully developed mixed convection in a parallel-plate vertical channel and compared it to their numerical results for developing flow at great distances from the channel entry. Cheng et al. [2] have investigated the problem of flow reversal and heat transfer of fully developed mixed convection in vertical channels. Analytical solution for fully developed mixed convection between parallel vertical plates with heat and mass transfer presented by Boulama and Galanis [3]. Rao and Narasimham [4] have considered the laminar conjugate mixed convection in a vertical channel with heat generating components. Ameni et al. [5] investigated numerically the mixed convection in a vertical heated channel.

A combined free and forced convection flow of an electrically conducting fluid in a channel in the presence of a transverse magnetic field is of special technical significance because of its frequent occurrence in many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, thermal insulation, petroleum reservoirs, etc. This type of problem also arises in electronic packages, microelectronic devices during their operations. In recent years, several convection heat transfer and fluid flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available convection heat transfer and fluid flow problems to include the effects of magnetic fields for those cases when the fluid is electrically conducting. Alireza and Sahai [6] studied the effect of temperature-dependent transport properties on the developing MHD flow and heat transfer in a parallel-plate channel

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whose walls were held at constant and equal temperatures. Umavathi and Malashetty [7] analyzed the problem of combined free and forced convective magnetohydrodynamic flow in a vertical channel by taking into account the effect of viscous and ohmic dissipations. Barletta and Ceilla [8] obtained the solutions both analytically by a power series method and numerically for Mixed convection MHD flow in a vertical channel by taking into account the effects of Joule heating and viscous dissipation. Recently Prathap Kumar et al. [9] studied mixed convection of composite porous medium in a vertical channel with a symmetric wall heating conditions.

In most of the MHD flow problems, the Hall and Ion-slip terms in Ohms law were ignored. However, in the presence of strong magnetic field, the influence of Hall current and Ion-slip are important. Tani [10] studied the Hall effects on the steady motion of electrically conducting viscous fluid in channels. Hall and Ion-slip effects on MHD Couette flow with heat transfer have been considered by Soundelgekar et al. [11]. Attia [12] considered the steady Couette flow of an electrically conducting viscous incompressible fluid between two parallel horizontal non-conducting porous plates with heat transfer, taking the Ion-slip into consideration.

During recent years the study of convection heat and mass transfer in non-Newtonian fluids has received much attention and this is because the traditional Newtonian fluids cannot precisely describe the characteristics of the real fluids. A theoretical study of the fully developed mixed convection flow of a micropolar fluid in a parallel plate vertical channel with an asymmetric wall temperature distribution has been presented by Ali and Chamkha [13]. Ziabakhsh and Domairry [14] have obtained the solution for natural convection of the Rivlin–Ericksen fluid of grade three between two infinite parallel vertical flat plates. Sajid et al. [15] studied fully developed mixed convection flow of a viscoelastic fluid between permeable parallel vertical walls using HAM. Some of the published papers on different non-Newtonian fluids, such as Brian [16], Kaloni and Siddiqui [17], Rudraiah et al. [18], Kaloni and Lou [19], Hayat et al. [20–22], Fetecau et al. [23–25]. In addition, progress has been considerably made in the study heat and mass transfer in magneto hydrodynamic flow of non-Newtonian fluids due to its application in many devices, like the MHD power generator, aerodynamics heating, electrostatic precipitation and Hall accelerator etc. Different models have been proposed to explain the behavior of non-Newtonian fluids. Among these, couple stress fluids introduced by Stokes [26] have distinct features, such as the presence of couple stresses, body couples and non-symmetric stress tensor. The couple stress fluid theory presents models for fluids whose microstructure is mechanically significant. The effect of very small microstructure in a fluid can be felt if the characteristic geometric dimension of the problem considered is of the same order of magnitude as the size of the microstructure. The main feature of couple stresses is to introduce a size dependent effect. Classical continuum mechanics neglects the size effect of material particles within the continua. This is consistent with ignoring the rotational interaction among particles, which results in symmetry of the force-stress tensor. However, in some important cases such as fluid flow with suspended particles, this cannot be true and a size dependent couple-stress theory is needed. The spin field due to microrotation of freely suspended particles set up an antisymetric stress, known as couple-stress, and thus forming couple-stress fluid. These fluids are capable of describing various types of lubricants, blood, suspension fluids etc. The study of couple-stress fluids has applications in a number of processes that occur in industry such as the extrusion of polymer fluids, solidification of liquid crystals, cooling of metallic plate in a bath, and colloidal solutions etc. Stokes [26] discussed the hydromagnetic steady flow of a fluid with couple stress effects. A review of couple stress (polar) fluid dynamics was reported by Stokes [27].

The homotopy analysis method [28] was first proposed by Liao in 1992, is one of the most efficient methods in solving different types of nonlinear equations such as coupled, decoupled, homogeneous and non-homogeneous. Also, HAM provides us a great freedom to choose different base functions to express solutions of a nonlinear problem [29]. The application of the homotopy analysis method (HAM) in engineering problems is highly considered by scientists, because HAM provides us with a convenient way to control the convergence of approximation series, which is a fundamental qualitative difference in analysis between HAM and other methods. Later Liao [30] presented an optimal homotopy analysis approach for strongly nonlinear differential equations. HAM is used to get analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation by Rashidi et al. [31]. Si et al. [32] accessed HAM solutions for the asymmetric laminar flow in a porous channel with expanding or contracting walls. Recent developments of HAM, like convergence of HAM solution, Optimality of convergence control parameter discussed by Turkyilmazoglu [33,34].

In this paper, we have investigated the Hall and Ion-slip effects on steady mixed convective heat transfer flow between two vertical parallel plates in couple stress fluid. The homotopy analysis method is employed to solve the governing nonlinear equations. Convergence of the derived series solution is analyzed. The behavior of emerging flow parameters on the velocity and temperature is discussed.

2. Mathematical formulation

Consider an incompressible electrically conducting couple stress fluid flow between two vertical parallel plates distance $2d$ apart. Choose the coordinate system such that x-axis be taken along vertically upward direction through the central line of the channel, y is perpendicular to the plates and the two plates are infinitely extended in the direction of x and z. The plates of the channel are at $y = \pm d$. The flow is subjected to a uniform magnetic field perpendicular to the flow direction with the Hall and Ion-slip effects. The effect of Hall and Ion-slip current gives rise to force in the z-direction, which induces a cross flow in that direction and hence the flow becomes three dimensional. Assume that the flow is steady and the magnetic Reynolds number is very small so that the induced magnetic field can be neglected in comparison with the applied magnetic