Commun Nonlinear Sci Numer Simulat 17 (2012) 2617–2627

Contents lists available at SciVerse ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

Anti-control of continuous-time dynamical systems

Simin Yu [a], Guanrong Chen [b,*]

[a] College of Automation, Guangdong University of Technology, Guangzhou 510006, China
[b] Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, China

ARTICLE INFO

Article history:
Received 3 July 2011
Received in revised form 2 October 2011
Accepted 2 October 2011
Available online 19 October 2011

Keywords:
Chaos
Continuous-time system
Anti-control
Global boundedness
Lyapunov exponent placement

ABSTRACT

Based on two basic characteristics of continuous-time autonomous chaotic systems, namely being globally bounded while having a positive Lyapunov exponent, this paper develops a universal and practical anti-control approach to design a general continuous-time autonomous chaotic system via Lyapunov exponent placement. This self-unified approach is verified by mathematical analysis and validated by several typical systems designs with simulations. Compared to the common trial-and-error methods, this approach is semi-analytical with feasible guidelines for design and implementation. Finally, using the Shilnikov criteria, it is proved that the new approach yields a heteroclinic orbit in a three-dimensional autonomous system, therefore the resulting system is indeed chaotic in the sense of Shilnikov.

1. Introduction

Anti-control of chaos, or chaotification, refers to the desire of generating chaos from a non-chaotic system by a simple control input. For continuous-time dynamical systems, although several successful techniques have been developed for the task, such as time-delay feedback, topological conjugate mapping, and impulsive control [1–10], there are no very effective and universal methodologies available in the literature today. Most reports in the existing literature took a trial-and-error approach to anti-controlling continuous-time autonomous systems, through parameter tuning, numerical simulation and Lyapunov exponent calculation, which by no means provide unified theoretical guidelines for designers to follow [11–13].

For discrete-time systems, namely for mappings, the situation is much more promising. Anti-control of discrete-time systems has developed several relatively complete theories and relatively mature techniques with analytic guidelines for the users supported by rigorous mathematical chaos theory [14–25]. As a result, the traditional numerical approach of trial-and-error with computer simulation has literally become only a means of verification in general. The first milestone of a mathematically-rigorous anti-control theory and method was attributed to the Chen–Lai anti-control algorithm initiated in 1996 [14–21], followed by the Wang–Chen chaotification scheme and the Shi–Chen theory of coupled-expanding maps developed in the 2000s [22–25].

By comparison, it is very natural to ask whether or not the discrete-time anti-control methods can be directly modified and applied to the continuous-time setting. The answer is generally *no*, because they are described by difference and differential equations respectively, which have many essential distinctions. One prominent difference in point is the Lyapunov exponent placement: a discrete chaotic system can have all positive Lyapunov exponents but a continuous counterpart typically needs to have positive, zero and negative Lyapunov exponents so as to stretch and fold the orbit flows in the phase space. Nevertheless, they still share many similarities and analogies in both system structure and dynamics.

* Corresponding author.
E-mail address: gchen@ee.cityu.edu.hk (G. Chen).

1007-5704/$ - see front matter © 2011 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2011.10.001

In this paper, based on the aforementioned two basic characteristics of continuous-time autonomous chaotic systems, namely being global bounded while having a positive Lyapunov exponent, a universal feedback controller design criterion is derived for anti-controlling continuous-time autonomous dynamical systems to become chaotic. First, general forms of uncontrolled and controlled systems and the feedback controller to be used are established such that the controlled system outputs are globally bounded while the controlled system has positive, zero and negative Lyapunov exponents. These generic forms allow a systematic design and parameter determination of the anti-controlled system, overcoming the time-consuming and uncertain trial-and-error parameter tuning disadvantages. To this end, the Shilnikov criteria are applied to a three-dimensional autonomous system as an example to show that the resulting anti-control system possesses a heteroclinic orbit therefore is chaotic in the sense of Shilnikov, which means the existence of Smale horseshoes.

The rest of the paper is organized as follows. Section 2 describes the two questions to be investigated. Section 3 proposes the general criterion for anti-controlling continuous-time autonomous dynamical systems. Section 4 shows several design examples in general forms. Section 5 demonstrates by the Shilnikov criteria the existence of a heteroclinic orbit in the designed anti-controlled three-dimensional autonomous system, thereby proving the chaoticity of the resulting system. Section 6 concludes the investigation.

2. Problem statements

Consider an n-dimensional continuous-time linear autonomous system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} \tag{1}$$

where $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]^T$ with a real system matrix

$$\boldsymbol{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \tag{2}$$

In modern control theory, a basic technical problem is: assuming that the origin of the uncontrolled system (1) is an unstable equilibrium, design a linear feedback controller for the system such that the origin of the controlled system becomes asymptotically stable.

On the contrary, a basic problem of anti-control theory is: assuming that the origin of the uncontrolled system (1) is an asymptotically stable equilibrium, design a simple nonlinear feedback controller $\boldsymbol{f}(\boldsymbol{\sigma x}, \boldsymbol{\varepsilon})$ such that the controlled system

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{f}(\boldsymbol{\sigma x}, \boldsymbol{\varepsilon}) \tag{3}$$

becomes chaotic, where $\boldsymbol{B}$ is a control matrix to be designed:

$$\boldsymbol{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{pmatrix} \tag{4}$$

and the nonlinear feedback controller

$$\boldsymbol{f}(\boldsymbol{\sigma x}, \boldsymbol{\varepsilon}) = \begin{pmatrix} f_1(\sigma_1 x_1, \varepsilon_1) \\ f_2(\sigma_2 x_2, \varepsilon_2) \\ \vdots \\ f_n(\sigma_n x_n, \varepsilon_n) \end{pmatrix} \tag{5}$$

where

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \end{pmatrix} \tag{6}$$

is the gain matrix, and $\boldsymbol{\varepsilon}$ is an upper bound for the controller (5), which is also to be designed:

$$\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]^T \tag{7}$$