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Optimization of three-loop missile autopilot gain under crossover frequency constraint

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Abstract

The open-loop crossover frequency is pointed as an important parameter for practical autopilot design. Since different gain designs may achieve the same open-loop crossover frequency, it should be neither considered as a performance objective of the optimal autopilot design-schemes nor neglected. Besides, the main assignment of the autopilot is to drive the missile to track the acceleration commands, so the autopilot gain design should be evaluated directly according to the resultant tracking performance. For this purpose, an optimal design methodology of the three-loop missile autopilot is introduced based on constraint optimization technique, where the tracking performance is established analytically as the design objective and the open-loop crossover frequency is formed as inequality constraint function, both are manipulated in terms of stable characteristic parameters of the autopilot closed-loop. The proposed technique is implemented with the assistance of a numerical optimization algorithm which automatically adjusts the design parameters. Finally, numerical simulation results are provided to demonstrate the effectiveness and feasibility of the proposed approach compared with that in some references.

Keywords: Three-loop missile autopilot; Optimal gain design; Crossover frequency constraint; Performance optimization

1. Introduction

The main mission of missile autopilot is to track the guidance commands with a guaranteed level of system performance. In order to successfully achieve this mission, the performance characteristics of the autopilot must have a fast response to intercept a maneuvering target and reasonable robustness for system stability under the effect of un-modeled dynamics and noise. Basically, the concept of open-loop transfer function is the cornerstone of feedback control system analysis, where the relative stability and the robustness can be determined from analysis of the stability margins. However, Ref. [1] shows that ignoring the value of open-loop crossover frequency in the design procedure, even with good phase and gain margins, will cause design instability for relatively innocuous plant perturbations. In fact, this design may cause too high crossover frequency, which indicates that the system may go unstable when it is built and tested. Moreover, Ref. [2] concludes that the concept of open-loop gain and phase margins is not as useful realistically at high frequency design due to the increase of model non-linearity, which leads to considerable difference between the predicted gain and phase values and their real values at high frequency. A common approach to address this problem is by modifying the crossover frequency value to make sure that the open-loop gain is below some desired level at high frequencies. This value is set based on the assumptions about the high-frequency modeling errors, sometimes based on test data, and often comes from hard-learned experience. A classical "rule of thumb" that addressed this value is introduced in Refs. [1,3]. As a result, the crossover frequency is an important parameter in gain design process to achieve good trade-off between fastness and robustness. Nevertheless, in multi-loop autopilot design different gain combinations could meet the same open-loop crossover frequency with different flight performances.

Consequently, different methods and strategies have been implemented by researchers in order to introduce the open-loop frequency requirements into the autopilot design procedure. From optimal design prospective, some methods are considered as weight adjusted LQR technique for the objective of minimum error between desired and actual open loop crossover frequency [4–7]. Although it is possible to get the same

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crossover frequency for different gain designs, these techniques take the prescribed crossover frequency as the design goal; this scheme will not essentially guarantee an ideal autopilot. Besides, it is based on initial guessing of weights which might need to be carried out and repeated many times to adjust the required initial performance. In a different way, the multi-objective optimization technique is introduced in Ref. [8], where both time and frequency performance design aspects are combined into one objective function through multi-weight technique. Even so, this method optimizes the autopilot to a certain specified performance level with the challenge of objective's weight adjustment. Moreover, Ref. [9] introduces a dynamic inversion technique, which uses a constraint optimization algorithm to get the design parameters for the autopilot system. However, this method is considered for a specified controller structure with some assumptions and totally numerical procedure. In addition, the system stability is confirmed by the inequality constraints on gain and phase margins and minimum controller cycle, which may perform a hard optimization problem with some performance degradation.

Since the crossover dynamic itself cannot determine the total performance of the autopilot, so it is necessary to find more reasonable objective function for optimal autopilot gain design so that an appropriate optimization technique can consider directly the open-loop crossover frequency constraint and avoid the burden of design weight adjustment. In this paper, an optimal autopilot gain design is introduced based on constraint optimization technique, where the tracking performance is set as design objective and the open-loop crossover frequency as design constraint. First, an analytical formula between the autopilot gain and the stable characteristic parameters of the autopilot closed loop is systematically derived. Then, the exact open-loop crossover frequency constraint is established in a form of analytical inequality. Moreover, the performance index ISE of the autopilot tracking error is analytically formed as the design objective [10–13]. Both the objective and the constraint are manipulated in terms of the characteristic parameters. Finally, a constrained optimization problem is constructed and the optimal gain design is achieved for the corresponding optimum design parameters with the assistance of an optimization algorithm. This work is extended to numerical autopilot design of a typical missile system using the proposed technique, and the results are compared with the design strategy of Ref. [14].

2. Missile modeling and analytical gain formula

The classic three-loop missile autopilot [15,16], namely Raytheon autopilot, depicted in Fig. 1, is the topology considered throughout this paper. Mathematically, the airframe transfer function input is the fin deflection, and the output is the achieved missile acceleration. The missile airframe dynamics is determined by six-dimensional equations of forces and moments acting on the missile body. The longitudinal missile dynamics, using the small disturbance linearization assumptions, are given as

$$\begin{aligned}
\ddot{\vartheta} &= M_\alpha \alpha + M_q \dot{\vartheta} + M_\delta \delta \\
\dot{\theta} &= -Z_\alpha \alpha - Z_\delta \delta \\
\alpha &= \vartheta - \theta \\
a_y &= -V\dot{\theta}
\end{aligned} \tag{1}$$

where ϑ is the body pitch angle, θ is the trajectory angle, α is the angle of attack, V is the missile velocity, δ is the fin deflection, a_y is the missile acceleration, and $M_\alpha, M_\delta, M_q, Z_\alpha$ and Z_δ are the aerodynamics coefficients [3]. The missile airframe transfer functions can be written as

$$\begin{aligned}
G_{\delta_c}^{a_y}(s) &= K_{q1}\frac{\left(1+T_1 s+T_2 s^2\right)}{\left(1+\dfrac{2\zeta_{AF}s}{\omega_{AF}}+\dfrac{s^2}{\omega_{AF}^2}\right)} \\
G_{\delta_c}^{\dot{\vartheta}}(s) &= K_{q3}\frac{\left(1+T_\alpha s\right)}{\left(1+\dfrac{2\zeta_{AF}s}{\omega_{AF}}+\dfrac{s^2}{\omega_{AF}^2}\right)}
\end{aligned} \tag{2}$$

where

$$\omega_{AF}^2 = -M_\alpha + Z_\alpha M_q, \quad \zeta_{AF} = \frac{-(Z_\alpha + M_q)}{2\omega_{AF}},$$

$$K_{q1} = \frac{-V_M\left(M_\alpha Z_\delta - M_\delta Z_\alpha\right)}{M_\alpha - Z_\alpha M_q}, \quad K_{q3} = \frac{K_{q1}}{V},$$

$$T_1 = \frac{Z_\delta M_q}{M_\alpha Z_\delta - M_\delta Z_\alpha}, \quad T_2 = \frac{Z_\delta}{M_\alpha Z_\delta - M_\delta Z_\alpha},$$

$$T_\alpha = \frac{M_\delta}{M_\alpha Z_\delta - M_\delta Z_\alpha}$$

The Raytheon autopilot is composed of rate loop, synthetic-stability loop and accelerometer feedback loop with feedback

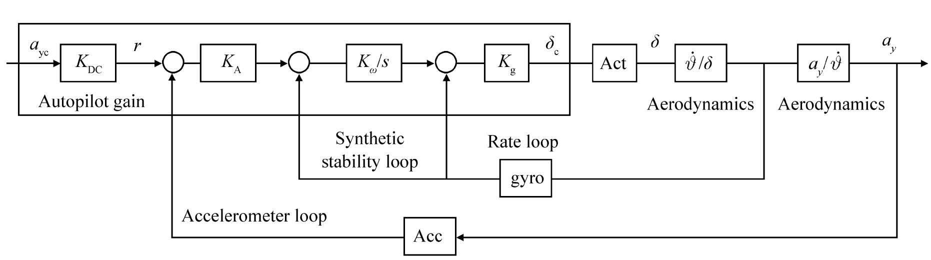

Fig. 1. Raytheon three-loop autopilot.