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Analytical solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient

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ABSTRACT

In this paper, the homotopy analysis method (HAM) is used to evaluate the analytical approximate solutions and efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient. The fin efficiency of the nonlinear fin problem with temperature-dependent thermal conductivity is obtained as a function of thermo-geometric fin parameter. It is shown that the thermal conductivity parameter has a strong influence over the fin efficiency. The analytic solution of the problem is obtained by using the HAM. The HAM contains the auxiliary parameter $\hbar$, which adjusts and controls the convergence region of the solution series in a simple way. By choosing the auxiliary parameter $\hbar$ in a suitable way, we can obtain reasonable solution for large values of M and β.

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1. Introduction

Fins are very frequently encountered in many engineering applications to enhance heat transfer. Numerous contributions have been made in the heat transfer analysis of the fins. The analytical solution can be easily obtained, while the thermal conductivity and the heat transfer coefficients are constant. If a large temperature difference exists within a fin, the thermal conductivity may not be constant. Furthermore, in general, the heat transfer coefficient may vary along a fin. The heat transfer coefficient may be a function of the spatial coordinate only along a fin or may depend on the local temperature difference between the fin surface and the surrounding fluid. The former is a typical problem for fins with a forced convection heat transfer, while the latter is common in other heat transfer modes such as natural convection, radiation, boiling and condensation heat transfer. A considerable amount of research has been conducted on the variable thermal conductivity and/or the non-uniform heat transfer coefficient. Due to the nonlinearity of the problem, in most cases numerical approaches are common but analytical approaches seem not to be tractable. The perturbation method [1,2] was used by Aziz and his co-workers for the convective fin with variable thermal conductivity. Pakdemirli and Sahin [3] and Bokhari et al. [4] introduced the symmetry method to obtain analytic solutions of an unsteady fin equation arising from temperature-dependent thermal

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Nomenclature

A_c	cross-sectional area of the fin
X	dimensional space coordinate
D_m	mth-order homotopy-derivative
h	heat transfer coefficient
K	dimensional thermal conductivity
k	non-dimensional thermal conductivity
L	length of the fin
$\mathscr{L}$	auxiliary linear operator
M	fin parameter defined in Eq. (3)
n	exponent in Eq. (2)
$\mathscr{N}$	nonlinear operator
P	perimeter of the fin
T	temperature
x	non-dimensional space coordinate
$\hbar$	auxiliary parameter
$H(\xi)$	auxiliary function
q	embedding parameter

Subscripts

a	surrounding fluid
b	base of the fin
c	tip end of the fin
m	order of approximation

Greeks

β	parameter in Eq. (3)
η	fin efficiency
θ	non-dimensional temperature
θ_m	mth-order approximation
χ_m	two-valued function
ξ	similarity variables

conductivity and a variable heat transfer coefficient. The Taylor series expansion method was applied to the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient by Kim and Huang [5]. The basic idea of homotopy in topology provided an idea to propose a general analytic method for nonlinear problems, namely homotopy analysis method (HAM), the method was proposed by Liao in 1992 [6–14]. This method is now widely used to solve many types of nonlinear problems. Many authors have used HAM to solve such problems [15–20]. We also use HAM for solving the fin problem. This is due to the fact that HAM contains the auxiliary parameter, and this parameter adjusts and controlls the convergence region of the solution series quite easily. In this paper, we introduce the basic idea of HAM and then we apply HAM to find an approximation of analytical solution of the nonlinear fin problem with temperature-dependent thermal conductivity and heat transfer coefficient.

2. Mathematical formulation

Here, we consider a straight one-dimensional fin with a constant cross-sectional area A_c. The fin, with perimeter P and length L, is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a. We also assume that the amount of heat transfer through the tip end is negligibly small. Bearing this in mind, the one-dimensional steady-state heat balance equation in dimensional form may be considered as

$$A_c \frac{d}{dX}\left(K\frac{dT}{dX}\right) - Ph(T - T_a) = 0, \quad 0 < X < L, \tag{1}$$

where h is the heat transfer coefficient and can be non-uniform along the fin. The heat transfer coefficient may depend on the temperature and usually can be expressed as a power form

$$h = h(T) = h_b\left(\frac{T - T_a}{T_b - T_b}\right)^n, \tag{2}$$

where h_b is the heat transfer coefficient at the base temperature. The exponent n depends on the heat transfer mode. Typical values of n are $-1/4$ for laminar film boiling or condensation, 1/4 for laminar natural convection, 1/3 for turbulent natural

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