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Three-dimensional flow of a Jeffery fluid over a linearly stretching sheet

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ABSTRACT

This investigation reports the three-dimensional flow of Jeffrey fluid over a linearly stretching surface. Transformation method has been utilized for the reduction of partial differential equations into the ordinary differential equations. The solutions of the nonlinear systems are presented by a homotopy analysis method (HAM). The reported graphical results are analyzed. A comparative study with the previous results of viscous fluid in the literature is made.

1. Introduction

Interest of the researchers in the flows of non-Newtonian fluids is on the leading edge during the last few decades. Such interest in fact is accelerated because of a broad range of applications of non-Newtonian fluids in the various disciplines, for instance in biological sciences, geophysics, chemical and petroleum industries. The Navier–Stokes equations cannot adequately describe the flow of non-Newtonian fluids. The constitutive equations are able to predict the rheological characteristics. In view of rheological parameters, the constitutive equations in the non-Newtonian fluids are more complex and thus give rise the equations which are complicated than the Navier–Stokes equations. The versatile nature of fluids does not provide a single constitutive equation by which all the non-Newtonian fluids can be studied. Hence several constitutive equations have been considered by the various researchers [1–10] in the field.

There is extensive literature available on the two-dimensional and axisymmetric flows over a stretching surface since the seminal works of Sakiadis [11,12]. The three-dimensional flow over a stretching surface has not been extensively discussed so far. Ariel [13] found the homotopy perturbation and exact solutions for the three-dimensional flow of a viscous fluid over a stretched surface. The magnetohydrodynamic (MHD) three-dimensional viscous flow over a porous stretching surface has been reported by Hayat and Javed [14]. Xu et al. [15] analyzed the MHD and heat transfer effects on the time-dependent three-dimensional flow over on impulsively stretching plate. Hayat and Awais [16] discussed the three-dimensional flow of a Maxwell fluid over a stretching surface.

The purpose of current investigation is to venture further in the regime of three-dimensional flows of the non-Newtonian fluids over a linearly stretching surface. Thus, we consider Jeffery fluid in this paper. The Jeffrey model [17–20] is relatively simpler linear model using time derivatives instead of convected derivatives for example the Maxwell model or an Oldroyd-B model does. This fluid model represents a rheology different from the Newtonian fluid. The paper is organized in the following pattern. Section 2 contains the formulation. The series solution by the homotopy analysis method (HAM) [21–26] has been developed and the related convergence analysis is presented in Section 3. The discussion regarding graphs is also included in the same section.

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2. Definition of the problem

We study an incompressible three-dimensional flow of a Jeffery fluid over a linearly stretching sheet at $z = 0$. The fluid occupies the space $z > 0$ and the motion of fluid is due to non-conducting stretching sheet. The constitutive expressions in a Jeffery fluid satisfy

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{1}$$

$$\mathbf{S} = \frac{\mu}{1+\lambda_1}(\dot{\mathbf{r}} + \lambda_2\ddot{\mathbf{r}}), \tag{2}$$

in which p denotes the pressure, $\mathbf{I}$ the identity tensor, μ the dynamic viscosity, λ_1 the ratio of relaxation and retardation times, λ_2 the retardation time, dots over the quantities denote material differentiation and

$$\dot{\mathbf{r}} = \nabla\mathbf{V} + (\nabla\mathbf{V})^T, \tag{3}$$

$$\ddot{\mathbf{r}} = \frac{d}{dt}(\dot{\mathbf{r}}), \tag{4}$$

where $\frac{d}{dt}$ is the material differentiation.

The continuity equation and equation of motion under the assumptions associated with the boundary layer flow yield

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 u}{\partial z^2} + \lambda_2\left(\frac{\partial u}{\partial z}\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial v}{\partial z}\frac{\partial^2 u}{\partial y \partial z} + \frac{\partial w}{\partial z}\frac{\partial^2 u}{\partial z^2} + u\frac{\partial^3 u}{\partial x \partial z^2} + v\frac{\partial^3 u}{\partial y \partial z^2} + w\frac{\partial^3 u}{\partial z^3}\right)\right], \tag{6}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \frac{\nu}{1+\lambda_1}\left[\frac{\partial^2 v}{\partial z^2} + \lambda_2\left(\frac{\partial u}{\partial z}\frac{\partial^2 v}{\partial x \partial z} + \frac{\partial v}{\partial z}\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial w}{\partial z}\frac{\partial^2 v}{\partial z^2} + u\frac{\partial^3 v}{\partial x \partial z^2} + v\frac{\partial^3 v}{\partial y \partial z^2} + w\frac{\partial^3 v}{\partial z^3}\right)\right], \tag{7}$$

with the following boundary conditions

$$\begin{gathered} u = u_w(x) = ax, \quad v = v_w(y) = by, \quad w = 0 \text{ at } z = 0, \\ u \to 0, \quad v \to 0, \quad \frac{\partial u}{\partial z} \to 0, \quad \frac{\partial v}{\partial z} \to 0, \text{ as } z \to \infty, \end{gathered} \tag{8}$$

where u, v and w are the velocities in the x, y and z directions, respectively, $\nu$the kinematic viscosity and the constants $a > 0$ and $b > 0$.

If prime denotes differentiation with respect to η then setting

$$\eta = \sqrt{\frac{a}{\nu}}z, \quad u = axf'(\eta), \quad v = ayg'(\eta), \quad w = -\sqrt{a\nu}\{f(\eta) + g(\eta)\} \tag{9}$$

Eq. (1) is automatically satisfied and Eqs. (5)–(8) give

$$f''' + (1+\lambda_1)\left[(f+g)f'' - f'^2\right] + \beta\left[f''^2 - (f+g)f'''' - g'f'''\right] = 0, \tag{10}$$

$$g''' + (1+\lambda_1)\left[(f+g)g'' - g'^2\right] + \beta\left[g''^2 - (f+g)g'''' - f'g'''\right] = 0, \tag{11}$$

$$\begin{gathered} f(0) = 0, \quad g(0) = 0, \quad f'(0) = 1, \quad g'(0) = c, \text{ at } \eta = 0, \\ f'(\infty) = 0, \quad g'(\infty) = 0, \quad f''(\infty) = 0, \quad g''(\infty) = 0, \text{ as } \eta \to \infty, \end{gathered} \tag{12}$$

in which the Deborah number β and the stretching ratio c are defined by

$$\beta = \lambda_2 a, \quad c = b/a. \tag{13}$$

It is noticed that the two-dimensional system ($g = 0$) can be recovered for $c = 0$ and is given by

$$f''' + (1+\lambda_1)\left[ff'' - f'^2\right] + \beta\left[f''^2 - ff''''\right] = 0, \tag{14}$$

$$\begin{gathered} f(0) = 0, \quad f'(0) = 1, \text{ at } \eta = 0, \\ f'(\infty) = 0, \quad f''(\infty) = 0 \text{ as } \eta \to \infty. \end{gathered} \tag{15}$$

For axisymmetric flow ($f = g$) and $c = 1$ and thus (10) reduces to

$$f''' + (1+\lambda_1)\left[2ff'' - f'^2\right] + \beta\left[f''^2 - 2ff'''' - f'f'''\right] = 0, \tag{16}$$

with the boundary conditions (15).