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Further advances on low-energy lunar impact dynamics

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ABSTRACT

We extend the analysis, started in a previous work [1], concerning the formation of lunar impact craters due to low-energy trajectories. First, we adopt the Circular Restricted Three-Body Problem and consider different choices of initial conditions inside the stable invariant manifold associated with the central invariant one in the neighborhood of the L_2 equilibrium point in the Earth–Moon system. Then we move to the Bicircular Restricted Four-Body Problem to study the effect of the Sun on the distribution of impacts on the Moon's surface.

1. Introduction

It is known that the dynamics induced by the invariant objects of the Circular Restricted Three-Body Problem (CR3BP) can help in the comprehension of Solar System natural transport phenomena: bodies such as comets and asteroids could be able to follow trajectories lying on or inside the hyperbolic invariant manifolds associated with periodic and quasi-periodic orbits around the collinear libration points. This kind of approach has been adopted in the past for example to explain the behavior of comets that are temporarily captured by Jupiter [14] or the origin of well-defined galactic structures [20,21].

Following this strand of research, in [1] we addressed the problem of low-energy trajectories that might have caused the formation of part of the impact craters that can be observed on the surface of the Moon. The phenomenon of craterization appears on all the rocky planets and satellites of the Solar System and it is widely studied by several branches of science, as it provides information on the target body and on the impactor in dynamical, astronomical and geological terms (see, for instance [12,16,19,22]).

As far as we know, the dynamics bringing lunar impacts has been always examined within a high-energy framework, being the average speed of encounter associated with the bolid at least 10 km/s. Instead, we mean to furnish a new dynamical insight on the collisional events that occurred on the Moon, by restricting the research to low-energy regimes, being the impact speed not greater than the lunar escape velocity (about 2.4 km/s). We notice that though nowadays it is hard to discover in the Earth–Moon neighborhood an object characterized by a low speed, i.e. low energy, at the epoch of the heavy lunar bombardment (between 4 and 3.8 billions years ago) the formation of the Solar System was not ended completely. Very likely minor bodies of different sizes and velocities were traveling around and this justifies the investigation we carry

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on. Also, on the Moon's surface they can be recognized craters of small diameter, that may be the result of low-energy impacts.

In the context of the CR3BP approximation, we introduced the idea of the stable invariant manifold associated with the central invariant manifold of the L_2 equilibrium point, namely $\mathcal{W}^s\left(\mathcal{W}^c_{L_2}\right)$, as impact gate for colliding asteroids in the Earth–Moon system and then we investigated the basics of the role of the Sun by means of the Bicircular Restricted Four-Body Problem (BR4BP). We showed that most of the collisions take place at the apex of the lunar surface (90°W,0°) if they are due to transit orbits uniformly distributed inside $\mathcal{W}^s\left(\mathcal{W}^c_{L_2}\right)$ and that there exists another main source of lunar impacts, represented by the dust, generated by high-energy collisions, that eventually can return to the Moon. Moreover, we detected that within the BR4BP model the initial phase corresponding to the position of the Sun can affect the percentage of impacts.

In this paper, we present some new advances that extend the just mentioned investigation. We analyze several distributions of transit trajectories inside $\mathcal{W}^s\left(\mathcal{W}^c_{L_2}\right)$ and we give further details on the behavior of the trajectories when accounting also for the Sun's gravitational influence. In particular, we select the initial conditions according either to the inclination or the semi-major axis of the osculating ellipse at the Earth and also we research the collisions due to an uniform distribution of initial conditions in a given energy level. The outcomes indicate that the impact concentration at the lunar apex persists, though in the last case is shifted somewhat. About the contribution of the Sun, we provide more details about the consequences drawn by the relative Earth–Moon–Sun configuration.

The paper is organized as follows. In Section 2 we recall the basic tools we take advantage of and the main properties of the dynamics under study. In Sections 3 and 4 we describe the explorations we perform assuming the CR3BP model and the related results. In Section 5 the simulations and outcomes obtained using the BR4BP.

2. Background

The Circular Restricted Three-Body Problem [23] studies the behavior of a particle P with infinitesimal mass m_3 moving under the gravitational attraction of two primaries P_1 and P_2, of masses m_1 and m_2, revolving around their center of mass on circular orbits.

The problem is usually tackled by adopting a synodical reference system, which is centered at the barycenter of the system and rotates around the z-axis with constant angular velocity equal to the mean motion of the primaries, and an adimensional set of units such that the gravitational constant, the sum of the masses of the primaries, the distance between them and the modulus of the angular velocity of the rotating frame are unitary. In this way m_1 and m_2 are fixed on the x-axis, as shown in Fig. 1 (left), and for the Earth–Moon system the unit of distance equals 384,400 km, the unit of time is 4.35 days and the dimensionless mass of the Moon is $\mu = \frac{m_2}{m_1+m_2} = 0.012150582$.

With these assumptions, the equations of motion can be written as

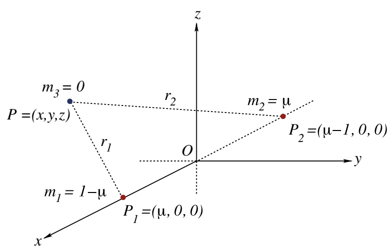
$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x}, \quad \ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y}, \quad \ddot{z} = \frac{\partial \Omega}{\partial z}, \tag{1}$$

where

$$\Omega(x,y,z) = \frac{1}{2}(x^2+y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{1}{2}(1-\mu)\mu, \tag{2}$$

and $r_1 = [(x-\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$ and $r_2 = [(x+1-\mu)^2 + y^2 + z^2]^{\frac{1}{2}}$ are the distances from P to P_1 and P_2, respectively.

It is known that in the synodical reference system there exist five equilibrium (or libration) points [23] (see Fig. 1, right) and that system (1) has a first integral, the *Jacobi integral*, which is given by



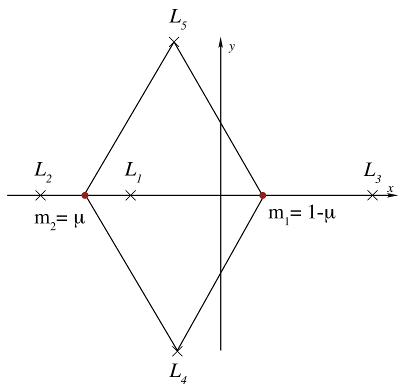


Fig. 1. Left: the Circular Restricted Three-Body problem in the synodical reference system with adimensional units. Right: the five equilibrium points associated with the problem.