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Dynamics of chemical wave segments with free ends

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Abstract

A quite simple but useful approach is performed for the analysis of chemical wave segments with free ends. By integrating a reaction–diffusion system we can obtain an analytical expression to understand the dynamics of the wave segments. This integration can yield qualitative information regarding wave development under an external forcing having feedback or noise effects. We conclude that this wave development is influenced not only by medium excitability but also by wave size.

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Keywords: Nonlinear dynamics; Reaction–diffusion pattern; Feedback stabilization

1. Introduction

Pattern formation such as target patterns and spiral waves with rotating free ends have been observed in a variety of dissipative systems. In living cells, for example, traveling action potentials on nerves, spiral waves of cyclic AMP on Dictyostelium, and so on, have been reported [1]. In chemical systems, the Belousov-Zhabotinsky (BZ) reaction [2–4] showed the formation of wavefronts of excitable domain and have been used to study the dynamics of spatiotemporal pattern formation. Recently, the dynamical responses of the pattern and/or the free end under external forcing with random fluctuations in excitable chemical systems have been investigated for the control of wave initiation and propagation [5–11]. Moreover various experiments regarding the BZ reaction with global feedback using a light effect as an external forcing have demonstrated a wider variety of spatiotemporal dynamics [12–15]. These experiments revealed that the size of the excitable domain, which is a chemical pattern, is an essential control parameter affecting the stability of pattern dynamics. These control problems in chemical systems were reviewed in details [16]. One of the authors has investigated the effect of a global feedback for propagating chemical wave segments with free ends [17–19]. These findings

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about the dynamics of spiral waves in excitable media have attracted the attention of an increasing number of researchers.

Our previous experimental and numerical studies have demonstrated the transition in which a wave of a certain size either expands or contracts at a particular excitability [17–19]. This was achieved by stabilizing the unstable steady state by means of a feedback method in the light-sensitive BZ reaction [20]. A numerical example of stabilized chemical wave segment is shown in Fig. 1(a). The segment having two free ends propagate while maintaining a stationary size and shape, and a constant velocity. To describe the dynamical behavior of the wave segment we use the following Oregonator model [21]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = D_u \nabla^2 u + \dfrac{1}{\varepsilon}\left(u - u^2 - (fv + \phi)\dfrac{u-q}{u+q}\right) & \text{in } \Omega, \\ \dfrac{\partial v}{\partial t} = D_v \nabla^2 v + u - v & \text{in } \Omega, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0 & \text{on } \partial\Omega, \\ u(\boldsymbol{r} = (x, y), 0) = u_0(\boldsymbol{r}), \quad v(\boldsymbol{r}, 0) = v_0(\boldsymbol{r}) & \text{in } \Omega. \end{cases} \tag{1}$$

Here, D_u, D_v, ε, f, and q are given positive constants, Ω is a domain with the boundary $\partial\Omega$, and ϕ is a control function which corresponds to the light intensity for the BZ reaction. To realize global feedback, the function ϕ is defined as

$$\phi(t) = a \int_\Omega u(\boldsymbol{r}, t)\, \mathrm{d}\boldsymbol{r} + b. \tag{2}$$

Here, we can consider $\int_\Omega u(\boldsymbol{r}, t)\, \mathrm{d}\boldsymbol{r}$ as the size of the excitable domain in which $u(\boldsymbol{r}, t)$ is larger than u_{th}. If $u(\boldsymbol{r}, t) < u_{th}$, we assume $u(\boldsymbol{r}, t) = q \approx 0$. The coefficients a and b are a gain positive constant and an offset constant, respectively.

Reaction–diffusion models such as the Oregonator model can involve the information of the size of the pattern in numerical calculation. To analyze for the 2D reaction–diffusion patterns with free ends directly reduces to solve the 2D free boundary problem. The general method to solve the problem have not been established, hence it give much difficulty in solving it. On the other hand, from an analytical viewpoint, the kinematic equation, which has some advantages in the analysis for the spatiotemporal patterns, was proposed [22,23]. The pattern with size on two dimensions is represented by a single smooth oriented curve in the kinematic system, hence, the analysis for the behavior of pattern is reduced to investigate an one-space-dimensional evolution equation. Some static approaches with the kinematic model have been presented for analyzing the stabilized

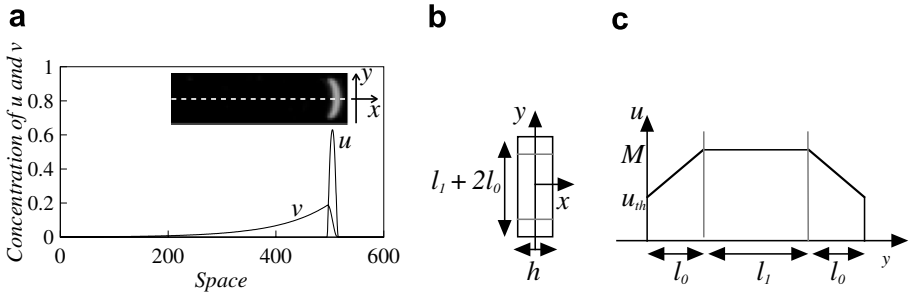

Fig. 1. (a) Numerical result of stabilized chemical wave segments, a picture in (a), and profiles of the concentrations of u and v along the white dotted line (x-axis) using System (1) with (2). A characteristic of a spiral wave in excitable media shows a critical behavior of the propagation, which sprouts or retracts when perturbed. It can, however, be stabilized by global feedback. Here we assume that the wave is rectangular in shape, (b), having a planar part and two free ends which is the tip of spiral wave with width h. And the profile of u along y-axis consists of a constant value M for the planar wave in region l_1 and a linear interpolative function for the tips in region l_0 (c). When $l_0 = 0.0$, the wave shows a 2D planar wave without the tip effect. Parameters of numerical results: $\varepsilon = 0.01$, $f = 2.5$, $q = 0.0002$, $D_u = 1.0$, $D_v = 0.0$, $a = 4.0$, and $b = -22.0$.