



Short communication

Response of Duffing system with delayed feedback control under combined harmonic and real noise excitations

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ABSTRACT

This paper presents a procedure for predicting the response of Duffing system with delayed feedback bang–bang control under combined harmonic and real noise excitations by using the stochastic averaging method. First, the time-delayed feedback bang–bang control force is expressed approximately in terms of the system state variables without time delay. Then the averaged Itô stochastic differential equations for the system are derived by using the stochastic averaging method. Finally, the response of the system is obtained by solving the Fokker–Plank–Kolmogorov (FPK) equation associated with the averaged Itô equations. It is shown that the time delay in feedback control can deteriorate the control effectiveness and cause bifurcation of stochastic jump of Duffing system. The validity of the proposed method is confirmed by digital simulation.

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1. Introduction

It is well known that time delay in real active control systems is usually unavoidable due to the time spent in measuring system states, on-line data processing, calculating and executing the control forces, etc. This time delay causes unsynchronized application of the control force, and often leads to poor performance or instability of the controlled systems. In the past two decades, much attention has been paid to studying the time delay problem in control community [1–3]. Time delay effects on controlled systems subject to deterministic excitation have been studied extensively [4–10]. For the case of controlled systems with time delay excited by Gaussian white noise, the effects of time delay on the controlled linear and nonlinear systems have been studied by Di Paola and coworkers [11,12] using an approach based on the Taylor expansion of the control force and another approach to finding exact stationary solution. Recently, the response and stability of quasi-integrable Hamiltonian systems with delayed feedback control under Gaussian white noise excitation have been studied by Liu and Zhu [13–15] using the stochastic averaging method.

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In all these studies, the excitation of systems is purely harmonic excitation or purely random noise. However, many mechanical and structural systems are often subjected to both random and harmonic excitations. A typical example of such systems is uncoupled flapping motion of rotor blades of a helicopter in forward flight under the effect of atmospheric turbulence [16]. To the author's knowledge, no attention has been paid to predicting the response of strongly nonlinear systems with delayed feedback control under combined harmonic and wide-band real noise excitations. In the present paper, the stochastic averaging method [17] is extended to predicting the response of strongly nonlinear Duffing system with delayed feedback bang–bang control under combined harmonic and wide-band real noise excitations. The delayed feedback bang–bang control force is expressed equivalently in terms of feedback bang–bang control force without time delay and the system is transformed into the Itô stochastic differential equations for the system without time delay, from which the averaged Itô equations are derived by using the stochastic averaging method and the stationary probability density of the amplitude and mean amplitude of the system are obtained from solving the Fokker–Plank–Kolmogorov (FPK) equation associated with the averaged Itô equations. The effect of time delay in feedback bang–bang control on the response is analyzed. Based on the stationary probability of amplitude obtained by the proposed procedure, the effect of time delay in feedback bang–bang control on the stochastic jump of Duffing oscillator is also studied.

2. Duffing system with delayed feedback bang–bang control

The motion equation of a nonlinearly damped Duffing oscillator with delayed feedback bang–bang control under external harmonic excitation and external and parametric wide-band noise excitations is of the form

$$\ddot{X} + (\beta_1 + \beta_2 X^2)\dot{X} + \omega_0^2 X + \alpha X^3 = E \cos \Omega t + \xi_1(t) + X \xi_2(t) + u_\tau, \quad (1)$$

where $\beta_1, \beta_2, \omega_0, \alpha, E, \Omega$ are positive constants denoting the damping coefficients, natural frequency of degenerated linear oscillator, intensity of nonlinearity, amplitude and frequency of harmonic excitation, respectively; $u_\tau = u(\dot{X}_\tau) = -b \operatorname{sgn}(\dot{X}(t - \tau))$ is time-delayed feedback bang–bang control force, where τ is the time delay, 'sgn' denotes sign function. u_τ has constant magnitude b , in the opposite direction of $\dot{X}_\tau$ and changes direction at $\dot{X}_\tau = 0$. $\xi_i(t)$ ($i = 1, 2$) are independently stationary and ergodic processes with zero mean and rational spectral densities

$$S_i(\omega) = \frac{D_i}{\pi} \frac{1}{(\omega^2 - \omega_i^2)^2 + 4\zeta_i^2 \omega_i^2 \omega^2}, \quad i = 1, 2 \quad (2)$$

in which ω_i, ζ_i and D_i are constants. $\xi_i(t)$ can be regarded as the output of the second order filter $\ddot{\xi}_i + 2\zeta_i \omega_i \dot{\xi}_i + \omega_i^2 \xi_i = W_i(t)$, ($i = 1, 2$) where $W_i(t)$ are Gaussian white noises in the sense of Stratonovich with intensities $2D_i$. It is assumed that β, E, b are of the same small order.

The response of system (1) can be considered as random spread of periodic motion of degenerated system $\ddot{x} + \omega_0^2 x + \alpha x^3 = 0$ around $(0, 0)$ in phase plane $(X, \dot{X})$. The solution can be assumed in the following form [17]:

$$\begin{aligned} X(t) &= A \cos \Phi(t) \\ \dot{X}(t) &= -A v(A, \Phi) \sin \Phi(t) \end{aligned} \quad (3)$$

where $\cos \Phi(t)$ and $\sin \Phi(t)$ are called the generalized harmonic functions,

$$\begin{aligned} \Phi(t) &= \Psi(t) + \Theta(t), \\ v(A, \Phi) &= \frac{d\Psi}{dt} = \sqrt{\frac{2[U(A) - U(A \cos \Phi)]}{A^2 \sin^2 \Phi}} \end{aligned} \quad (4)$$

and A, Φ, Ψ and Θ are all random processes. $U(x) = \omega_0^2 x^2/2 + \alpha x^4/4$ is the potential energy of the system. $v(A, \Phi)$ is instantaneous frequency of the system and can be approximated by the following finite sum with a relative error less than 0.03%:

$$v(a, \varphi) = v_0(a) + v_2(a) \cos 2\varphi + v_4(a) \cos 4\varphi + v_6(a) \cos 6\varphi, \quad (5)$$

where

$$\begin{aligned} v_0(a) &= (\omega_0^2 + 3\alpha a^2/4)^{1/2} (1 - \lambda^2/16), \\ v_2(a) &= (\omega_0^2 + 3\alpha a^2/4)^{1/2} (\lambda/2 + 3\lambda^3/64), \\ v_4(a) &= (\omega_0^2 + 3\alpha a^2/4)^{1/2} (-\lambda^2/16), \\ v_6(a) &= (\omega_0^2 + 3\alpha a^2/4)^{1/2} (\lambda^3/64), \\ \lambda &= \alpha a^2/4(\omega_0^2 + 3\alpha a^2/4). \end{aligned} \quad (6)$$

The average frequency $\omega(a) = v_0(a)$, and the following approximate relation can be used in the following averaging:

$$\Phi(t) \approx \omega(A)t + \Theta(t). \quad (7)$$

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