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A modified tanh–coth method for solving the general Burgers–Fisher and the Kuramoto–Sivashinsky equations

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ABSTRACT

In this work we use a modified tanh–coth method to solve the general Burgers–Fisher and the Kuramoto–Sivashinsky equations. The main idea is to take full advantage of the Riccati equation that the tanh-function satisfies. New multiple travelling wave solutions are obtained for the general Burgers–Fisher and the Kuramoto–Sivashinsky equations.

1. Introduction

In this work we establish new travelling wave solutions to the general Burgers–Fisher (BF) and the Kuramoto–Sivashinsky (KS) equations given by

$$u_t + au^n u_x + bu_{xx} + ku(1 - u^n) = 0, \tag{1}$$

and

$$u_t + auu_x + bu_{xx} + ku_{xxxx} = 0, \tag{2}$$

respectively, where a, b and k are some arbitrary constants. The general Burgers–Fisher equation (1), see [1,2], has a wide range of applications in plasma physics, fluid physics, capillary-gravity waves, nonlinear optics and chemical physics [3].

The Kuramoto–Sivashinsky equation (2) models the fluctuations of the position of a flame front, the motion of a fluid going down a vertical wall, or a spatially uniform oscillating chemical reaction in a homogeneous medium [4]. It has also been examined as a prototypical example of spatiotemporal chaos in one space dimension [5]. In addition, this equation was originally derived in the context of plasma instabilities, flame front propagation, and phase turbulence in reaction–diffusion system [5].

Finding exact solutions of nonlinear partial differential equations (PDE's) has become more attractive subject due to the widespread of computer algebraic system (CAS), such as Maple and Mathematica. CAS allows us to do tedious and lengthy manipulations. Moreover, CAS can help us find new exact solutions of nonlinear PDE's.

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Many methods were used to obtain travelling solitary wave solutions to nonlinear PDE's, such as the inverse scattering method [6–8], Hirota's bilinear method [9,10], the tanh method [11,12], the sine–cosine method [13,14], Backlund transformation method [15,16], the homogeneous balance [17,18], Darboux transformation [19], the Jacobi elliptic function expansion method [20].

Among those, the tanh method, established by Malfliet [11], uses a particularly straightforward and effective algorithm to obtain solutions for a large numbers of nonlinear PDE's. In recent years, much research work has been concentrated on the various extensions and applications of the tanh method. Fan [21,22] has proposed an extended tanh method and obtained new traveling wave solutions that cannot be obtained by the tanh method. Recently, Wazwaz extended the tanh method and call it first the extended tanh method [23–25] and later as the tanh–coth method [26]. Most recently, El-Wakil [27,28] and Soliman [29] modified the extended tanh method (the tanh–coth method) and obtained new solutions for some nonlinear PDE's. The goal of this work is to implement the tanh–coth method and the Riccati equation in [30] to obtain more new exact travelling wave solutions of the general BF and the KS equations.

2. Description of the method

Consider the general nonlinear wave PDE's, say, in two variables

$$u_t = G(u, u_x, u_{xx}, \ldots). \tag{3}$$

In order to apply the tanh–coth method, the independent variables, x and t, are combined into a new variable, $\xi = \mu(x - ct)$, where μ and c are undetermined parameters which represent the wave number and velocity of the traveling wave, respectively. Therefore, $u(x,t)$ is replaced by $u(\xi)$, which defines the traveling wave solutions of (3). Equations such as (3) are then transformed into

$$-\mu c \frac{du}{d\xi} = G\left(u, \mu \frac{du}{d\xi}, \mu^2 \frac{d^2u}{d\xi^2}, \ldots\right). \tag{4}$$

Hence, under the transformation $\xi = \mu(x - ct)$, the PDE in (3) has been reduced to an ordinary differential equation (ODE) given by (4). The resulting ODE is then solved by the tanh–coth method [25], which admits the use of a finite series of functions of the form

$$u(x,t) = u(\xi) = a_0 + \sum_{j=1}^{M} \left[a_j Y^j(\xi) + b_j Y^{-j}(\xi)\right], \tag{5}$$

and the Riccati equation

$$Y' = \alpha + \beta Y + \gamma Y^2, \tag{6}$$

where $' := \frac{d}{d\xi}$, and α, β, and γ are constants to be prescribed later. The parameter M is a positive constant that can be determined by balancing the linear term of highest order with the nonlinear term in (4). Inserting (5) into the ODE in (4) and using (6) results in an algebraic equation in powers of Y. Since all coefficients of Y^j must vanish. This will give a system of algebraic equations with respect to parameters a_i, b_i, μ and c. With the aid of Maple, we can determine a_i, b_i, μ and c.

We will consider the following special solutions of the Riccati equation (6)

(I) $\alpha = \beta = 1$ and $\gamma = 0$, (6) has the solution $Y = e^{\xi} - 1$.
(II) $\alpha = \gamma = \pm 1/2$ and $\beta = 0$, (6) has the solutions $Y = \sec\xi \pm \tan\xi$ and $Y = \csc\xi \mp \cot\xi$, respectively.

To illustrate the method, we consider the general BF and the KS equations below.

3. Applications

3.1. The general Burgers–Fisher equation

Let us first consider the general BF equation which has the form

$$u_t + au^n u_x + bu_{xx} + ku(1 - u^n) = 0. \tag{7}$$

In order to obtain travelling wave solutions for Eq. (7), we use

$$u(x,t) = u(\xi), \quad \xi = \mu(x - ct). \tag{8}$$

Substituting (8) into (7), we obtain

$$-cu' + au^n u'' + bu'' + ku(1 - u^n) = 0. \tag{9}$$

Balancing the order of the nonlinear term $u^n u'$ with the linear term u'' in (9), we obtain