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Synchronization of Chua's circuits with multi-scroll attractors: Application to communication

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ABSTRACT

In this paper, the synchronization problem of coupled Chua's circuits generator of *n*-scroll chaotic attractors in master–slave configuration is numerically studied. In particular, we consider a modified Chua's circuit generator of 5-scroll chaotic attractors by using Hamiltonian systems and state observer approach. A potential application to transmit encrypted audio and image information is also given.

1. Introduction

The great variety of the so-called multi-scroll chaotic attractors have their origin in the double scroll chaotic attractor, which is generated by the well-known Chua's circuit [1,2]. Several generalizations of Chua's circuit lead to multi-scroll attractors. Generation of multi-scroll chaotic attractors have received considerable attention for more than one decade, it is a topic of both theoretical and practical interests [3–13]. In addition, because of their potential application in communications, cryptography, and neural networks [5] by using Chua's circuit generator of multi-scroll attractors.

The main generalizations that have been reported in the current literature to the Chua's circuit, are: (i) the modification of the nonlinear characteristic, and (ii) the increment in the space dimension of the circuit. For example in [6,7] a family of the so-called *n*-double scroll attractors was introduced. This was done by adding breakpoints in the nonlinear characteristic of Chua's diode. A more complete family of *n*-scroll chaotic attractors instead of *n*-double scroll chaotic attractors, which generates an odd number of scrolls instead of an even number of scroll attractors, has been obtained from the generalized Chua's

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circuit in [8]. The literature on the generation of multi-scroll chaotic attractors is abundant, see e.g., [3–16]. In particular, in [3] was confirmed experimentally the 3- and 5-scroll chaotic attractors in a generalized Chua's circuit, and in [4] was proposed a systematical circuitry design method to realized physically up to 10-scrolls.

On the other hand, in the past decades, the chaotic synchronization problem has received a tremendous increasing interest see e.g., [17–28] and references therein. This property is supposed to have interesting applications in different fields, particularly to design private/secure communication systems. The broadband and noise-like characteristic of chaotic signals are seen as possibly highly secure media for communication [20,21,27,29–35]. The private/secure communication schemes are usually constituted by a chaotic system as transmitter along with an identical chaotic system as receiver; where the confidential information is imbedded into the transmitted chaotic signal by direct modulation, masking, or another technique. At the receiver end, if chaotic synchronization can be achieved, then it is possible to extract the hidden information from the transmitted signal.

The main goals of this paper are: (i) to obtain synchronization of two generalized Chua's circuits generator of multiple scroll chaotic attractors in master–slave configuration. This objective is achieved by appealing to Hamiltonian systems and state observer approach from nonlinear control theory [19]. And, (ii) to transmit encrypted audio and image messages based on chaos synchronization.

The organization of the paper is as follows: in Section 2, we give a brief review on synchronization of chaotic circuits via Hamiltonian systems and observer approach. In Section 3, a mathematical model of modified Chua's circuit generator of multi-scroll chaotic attractors is introduced. In Section 4, we show chaos synchronization of Chua's circuits generator of *n*-scroll chaotic attractors. In Section 5, we carry out a stability analysis of the synchronization error. In Section 6, we present an application to chaotic communications of confidential information. Finally, some conclusions are given in Section 7.

2. Review on chaos synchronization via Hamiltonian forms and observer approach

Consider the following dynamical system:

$$\dot{x} = f(x), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function.

In [19] is reported how the dynamical system (1) can be written in the following *Generalized Hamiltonian canonical form*:

$$\dot{x} = \mathscr{J}(x)\frac{\partial H}{\partial x} + \mathscr{S}(x)\frac{\partial H}{\partial x} + \mathscr{F}(x), \quad x \in \mathbb{R}^n, \tag{2}$$

$H(x)$ denotes a smooth *energy function* which is globally positive definite in $\mathbb{R}^n$. The *gradient vector* of H, denoted by $\partial H/\partial x$, is assumed to exist everywhere. We use *quadratic* energy function $H(x) = (1/2)x^T\mathscr{M}x$ with $\mathscr{M}$ being a, constant, symmetric positive definite matrix. In such case, $\partial H/\partial x = \mathscr{M}x$. The matrices, $\mathscr{J}(x)$ and $\mathscr{S}(x)$ satisfy, for all $x \in \mathbb{R}^n$, the properties: $\mathscr{J}(x) + \mathscr{J}^T(x) = 0$ and $\mathscr{S}(x) = \mathscr{S}^T(x)$. The vector field $\mathscr{J}(x)\partial H/\partial x$ exhibits the conservative part of the system and it is also referred to as the workless part, or work-less forces of the system; and $\mathscr{S}(x)$ depicting the working or nonconservative part of the system. For certain systems, $\mathscr{S}(x)$ is negative definite or negative semidefinite. Thus, the vector field is considered as the dissipative part of the system. If, on the other hand, $\mathscr{S}(x)$ is positive definite, positive semidefinite, or indefinite, it clearly represents, respectively, the global, semi-global, and local destabilizing part of the system. In the last case, we can always (although nonuniquely) decompose such an indefinite symmetric matrix into the sum of a symmetric negative semidefinite. matrix $\mathscr{R}(x)$ and a symmetric positive semidefinite matrix $\mathscr{N}(x)$. Finally, $\mathscr{F}(x)$ represents a locally destabilizing vector field.

In the context of observer design, we consider a *special class* of Generalized Hamiltonian forms with linear output map $y(t)$, given by

$$\dot{x} = \mathscr{J}(y)\frac{\partial H}{\partial x} + (\mathscr{I} + \mathscr{S})\frac{\partial H}{\partial x} + \mathscr{F}(y), \quad x \in \mathbb{R}^n, \quad y = \mathscr{C}\frac{\partial H}{\partial x}, \quad y \in \mathbb{R}^m, \tag{3}$$

where $\mathscr{S}$ is a constant symmetric matrix, not necessarily of definite sign. The matrix $\mathscr{I}$ is a constant skew symmetric matrix, and $\mathscr{C}$ is a constant matrix.

We denote the *estimate* of the state $x(t)$ by $\xi(t)$, and consider the Hamiltonian energy function $H(\xi)$ to be the particularization of H in terms of $\xi(t)$. Similarly, we denote by $\eta(t)$ the estimated output, computed in terms of the estimated state $\xi(t)$. The gradient vector $\partial H(\xi)/\partial \xi$ is, naturally, of the form $\mathscr{M}\xi$ with $\mathscr{M}$ being a, constant, symmetric positive definite matrix.

A *nonlinear state observer* for the Generalized Hamiltonian form (3) is given by

$$\dot{\xi} = \mathscr{J}(y)\frac{\partial H}{\partial \xi} + (\mathscr{I} + \mathscr{S})\frac{\partial H}{\partial \xi} + \mathscr{F}(y) + K(y - \eta), \quad \xi \in \mathbb{R}^n, \quad \eta = \mathscr{C}\frac{\partial H}{\partial \xi}, \quad \eta \in \mathbb{R}^m, \tag{4}$$

where K is the *observer gain*.

The *state estimation error*, defined as $e(t) = x(t) - \xi(t)$ and the output estimation error, defined as $e_y(t) = y(t) - \eta(t)$, are governed by

$$\dot{e} = \mathscr{J}(y)\frac{\partial H}{\partial e} + (\mathscr{I} + \mathscr{S} - K\mathscr{C})\frac{\partial H}{\partial e}, \quad e \in \mathbb{R}^n, \quad e_y = \mathscr{C}\frac{\partial H}{\partial e}, \quad e_y \in \mathbb{R}^m, \tag{5}$$