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Stability analysis of uncertain fuzzy Hopfield neural networks with time delays

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ABSTRACT

In this paper, the global stability problem of uncertain Takagi–Sugeno (T–S) fuzzy Hopfield neural networks with time delays (TSFHNNs) is considered. A novel LMI-based stability criterion is obtained by using Lyapunov functional theory to guarantee the asymptotic stability of TSFHNNs. Here, we choose a generalized Lyapunov functional and introduce a parameterized model transformation with free weighting matrices to it, in order to obtain generalized stability region. In fact, these techniques lead to generalized and less conservative stability condition that guarantee the wide stability region. The proposed stability conditions are demonstrated with four numerical examples. Comparison with other stability conditions in the literature shows our conditions are the more powerful ones to guarantee the widest stability region.

1. Introduction

In recent years, the well-known Hopfield neural network has been extensively studied, and successfully applied in many areas such as combinatorial optimization, signal processing and pattern recognition, see e.g. [1,2]. Recently, it has been realized that significant time delays as a source of instability and bad performance may occur in neural processing and signal transmission. Thus, the stability problem of Hopfield neural networks has became interesting and many sufficient conditions have been proposed to guarantee the asymptotic or exponential stability for the neural networks with various type of time delays, see for example [3–10].

In practical systems, analysis of a mathematical model is usually an important work for a control engineer as to control a system. However, the mathematical model always contains some uncertain elements; these uncertainties may be due to additive unknown internal or external noise, environmental influence, poor plant knowledge, reduced-order models, uncertain or slowly varying parameters. Therefore, under such imperfect knowledge of the mathematical model, seeking

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to design a robust control such that the system responses can meet desired properties is an important topic in system theory. Hence, robust stability analysis for uncertain time delay systems have been the focus of much research in recent years [9,11–14].

FUZZY systems in the form of the Takagi–Sugeno (T–S) model [15] have attracted rapidly growing interest in recent years [16,17]. T–S fuzzy systems are nonlinear systems described by a set of IF–THEN rules. It has shown that the T–S model can give an effective way to represent complex nonlinear systems by some simple local linear dynamic systems with their linguistic description. Some nonlinear dynamic systems can be approximated by the overall fuzzy linear T–S models for the purpose of stability analysis [16,17]. Originally, Tanaka and his colleagues have provided a sufficient condition for the quadratic stability of the T–S fuzzy systems in the sense of Lyapunov in a series of papers [18,19] by considering a Lyapunov function of the sub-fuzzy systems of the T–S fuzzy systems. The concept of incorporating fuzzy logic into a neural network is proposed in some papers [20–25].

Based on the above discussions, we shall generalize the ordinary T–S fuzzy models to express a class of Hopfield neural network with time delays. The main purpose of this paper is to study the global stability results of TSFHNNs in terms of LMIs. The main advantage of the LMI-based approaches is that the LMI stability conditions can be solved numerically using MATLAB LMI toolbox [26] which implements the state of art interior-point algorithms [27]. We also provide a numerical example to demonstrate the effectiveness of the proposed stability results.

2. System description and preliminaries

Consider the following uncertain Hopfield neural networks with time delays described by:

$$\dot{x}(t) = -(A + \Delta A)x(t) + (W + \Delta W)f(x(t-\tau)), \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbf{R}^n$ is the neural state vector, τ is the unknown time delay. The matrices $A = diag\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix and $a_i > 0, i = 1, \ldots, n, W \in \mathbf{R}^{n\times n}$ are the connection weight matrices, ΔA and ΔW represent the parameter uncertainties. $f(x(t)) = [f_1(x_1(\cdot)), f_2(x_2(\cdot)), \ldots, f_n(x_n(\cdot))]^T \in \mathbf{R}^n$ is the neuron activation function with $f(0) = 0$.

The initial condition associated with model (1) is

$$x_i(s) = \phi_i(s), \quad i = 1, 2, \ldots, n,$$

where $\phi_i(s)$ is bounded and continuously differential on $[-\tau, 0]$.

Throughout this paper, we make the following assumption:

(**A**) There exist positive numbers L_q such that

$$0 \leqslant \frac{f(x) - f(y)}{x - y} \leqslant L_q, \quad q = 1, 2, \ldots, n,$$

for all $x, y \in R,\ x \neq y$ and denote $L = diag\{L_1, L_2, \ldots, L_n\}$.

Suppose (**A**) holds, then it is clear that the conditions of Lemma 2 in [28] hold for the functions $f(\cdot)$. Therefore, similarly to the proof of Theorem 1 in [28], we can obtain that the equilibrium point of the system Eq. (1) is exist and unique.

In this section, we will consider a Hopfield neural network with time delays, which is represented by a T–S fuzzy model composed of a set of fuzzy implications and each implications is expressed as a linear system model.

The continuous fuzzy system was proposed to represent a nonlinear system [15]. The system dynamics can be captured by a set of fuzzy rules which characterize local correlation in the state space. Each local dynamic described by the fuzzy IF–THEN rule has the property of linear input–output relation. Based on the T–S fuzzy model concept, a general class of T–S fuzzy Hopfield neural networks with time delays is considered here. The model of Takagi–Sugeno fuzzy Hopfield NNs with time delays is described as follows.

Plant Rule k:

IF $\{\theta_1(t)\ is\ M_{k1}\}$ and $\ldots$ and $\{\theta_r(t)\ is\ M_{kr}\}$

THEN

$$\dot{x}(t) = -(A_k + \Delta A_k)x(t) + (W_k + \Delta W_k)f(x(t-\tau)), \tag{2}$$

where $\theta_i(t), (i = 1, 2, \ldots, r)$ are known variables. $M_{kl}(k \in \{1, 2, \ldots, m\}, l \in \{1, 2, \ldots, r\})$ is the fuzzy set and m is the number of model rules. The parameter uncertainties $\Delta A_k,\ \Delta W_k$ are time varying matrices with appropriate dimensions, which are defined as follows:

$$[\Delta A_k\ \Delta W_k] = M_k F(t)[E_{1k}\ E_{2k}],$$

where E_{1k}, E_{2k}, M_k are known constant matrices of appropriate dimensions and $F(t)$ is an known time varying matrix with Lebegue measurable elements bounded by

$$F^T(t)F(t) \leqslant I,$$

where I is the identity matrix with appropriate dimension.