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Analytical solution in 2D domain for nonlinear response of piezoelectric slabs under weak electric fields

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Piezoceramic materials exhibit different types of nonlinearities depending upon the magnitude of the mechanical and electric field strength in the continuum. Some of the nonlinearities observed under weak electric fields are: presence of superharmonics in the response spectra and jump phenomena etc. especially if the system is excited near resonance. In this paper, an analytical solution (in 2D plane stress domain) for the nonlinear response of a rectangular piezoceramic slab has been obtained by use of Rayleigh–Ritz method and perturbation technique. The eigenfunction obtained from solution of the differential equation of the linear problem has been used as the shape function in the Rayleigh–Ritz method. Forced vibration experiments have been conducted on a rectangular piezoceramic slab by applying varying electric field strengths across the thickness and the results have been compared with those of analytical solution. The analytical solutions compare well with those of experimental results. These solutions should serve as a method to validate the FE formulations as well as help in the determination of nonlinear material property coefficients for these materials.

1. Introduction

With a number of piezoelectric finite element models existing in the literature to cope with complicated boundary value problems of smart structures, it is very important to obtain analytical solutions for a few simple geometries for verification of finite element formulations. Moreover, it is also necessary to develop analytical solutions for simple geometry and loading conditions so that these can be used for comparison with experimental results. By optimizing the analytical solutions with experimental results, the unknown material properties can be determined.

The coupling of mechanical and electrical properties of piezoelectric materials makes the analytical solution complicated. Piezoelectric plates composed of orthotropic or anisotropic layers are usually analyzed using laminated composite plate theory. For the analytical solution of these plates, Pagano [8] first presented the exact solution of a laminated plate under cylindrical bending. Ray et al. [10–12], Heyliger and Brooks [3] extended this methodology to develop the exact solutions for the piezoelectric laminated plates with the help of Eshelby–Stroh formalism. Taking the thermal effect into account, Xu et al. [21] presented a 3D analysis for the coupled thermo-electro-elastic response of multilayered hybrid composite plates with four

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simply supported edges by using the state space method. Dube et al. [2] developed a series solution method for a single layer thermo-electro-elastic plate subjected to cylindrical bending using the approach of Pagano [8].

To solve the dynamic problem of piezoelectric array elements, several simplified solutions for modeling the linear piezoelectric behaviour in 1D, 2D and 3D domains were tried. Ray et al. [13] presented an exact solution for analyzing dynamics of composite plates with piezoelectric layers bonded at the top and bottom surfaces. Later, Ray [14] also presented the closed-form solution for the optimal control of flexural vibration of a simply supported symmetric laminated plate. Wolf and Gottlieb [20] derived the nonlinear equations of motion for a cantilever beam covered by PZT layers in symmetric and asymmetric configuration. By defining a nonlinear enthalpy function with nonlinear strain and applying Hamilton's principle, they derived the equations of motion and solved by perturbation analysis and multiple scales method.

Beige and Schmidt [1] first observed the nonlinear phenomena under weak electric fields such as presence of superharmonics in the response spectra and jump phenomena etc. while conducting experiments on PZT materials. They modelled the nonlinearities under weak electric fields using higher order quadratic and cubic elastic terms in the electric enthalpy function. However, their models and solutions techniques are restricted to one-dimensional (1D) piezoelectric continuum. Later on, von Wagner and Hagedorn [18] studied the nonlinearity in a piezo-beam system. von Wagner [19] and Neumann [7] also predicted the amplitude of displacement and current responses of piezo-rods of different diameters using a 1D nonlinear formulation which used the nonlinear damping theory (for deriving the virtual work done by the dissipative damping forces). Samal et al. [16] developed a generalized 3D formulation for modeling the nonlinear behaviour of piezoelectric materials under weak electric fields using a 3D nonlinear electric enthalpy density function term and nonlinear dissipative work term δW_D in the extended Hamilton's principle. Samal et al. [16] also developed a finite element (FE) solution procedure for the above generalized nonlinear model and applied it to solve different problems in the 2D and 3D domain and compared the FE results with those of experiment [17].

In this paper, a closed form solution for displacement and current responses of a simple rectangular geometry has been developed in 2D domain based on the nonlinear electric enthalpy function and virtual work (due to nonlinear damping) of Samal et al. [16]. Experiments have been conducted on a rectangular slab of PIC 181 material and the results have been compared with those of closed form solution. The paper is organized in six sections. The first section gave a brief overview of the research works on analytical solution techniques for modeling piezoelectric material behaviour available in literature. Section 2 briefly describes the generalized nonlinear electric enthalpy function recently proposed by Samal et al. [16,17]. The third section describes the application of Hamilton principle to the nonlinear piezoelectric continuum whereas the fourth section describes the details of the analytical solution technique that is followed for a rectangular piezoelectric slab. The experimental results have been compared with the analytical solutions in fifth section and we conclude the paper with scope of future research in the sixth section.

2. Generalized nonlinear constitutive equation of the piezoelectric continuum

The electric enthalpy density function is generally used to derive the governing equations of the coupled piezoelectric continuum. For the linear piezoelectric behaviour in 2D domain (with S_{xx},S_{yy} and S_{xy} as the only non-zero components of strain tensor, the linear electric enthalpy function ($H = H_{lin}$) is given as [4]

$$H_{lin} = \frac{1}{2}c_{11}S_{xx}^2 + c_{12}S_{xx}S_{yy} + \frac{1}{2}c_{22}S_{yy}^2 + \frac{1}{2}c_{66}S_{xy}^2 - \gamma_{31}^{(0)}S_{xx}E_z - \gamma_{32}^{(0)}S_{yy}E_z - \frac{1}{2}\nu_0E_z^2 \tag{1}$$

In order to model the nonlinearities in a coupled piezoelectric medium, the linear electric enthalpy density function has been modified by incorporating higher order terms (viz cubic and fourth order terms in strain and electric fields) in the energy expression of the coupled piezoelectric domain. It may be noted the material properties are constant for the linear electric enthalpy function, whereas these can be described as functions of strain and electric field components in the nonlinear theory. Hence, the nonlinear electric enthalpy has been constituted by incorporating cubic and fourth order expressions of strain and electric field components multiplied with unknown material nonlinear constants such that these quantities represent energy in mechanical, dielectric and piezoelectric domains. The nonlinear electric enthalpy density function H_{nonl} is written as

$$\begin{aligned}
H_{nonl} =& \frac{1}{2}c_{11}S_{xx}^2 + c_{12}S_{xx}S_{yy} + \frac{1}{2}c_{22}S_{yy}^2 + \frac{1}{2}c_{66}S_{xy}^2 - \gamma_{31}^{(0)}S_{xx}E_z - \gamma_{32}^{(0)}S_{yy}E_z - \frac{1}{2}\nu_0E_z^2 \\
&+ \frac{1}{3}c_{11}^{(1)}S_{xx}^3 + \frac{1}{3}c_{12}^{(1)}S_{xx}S_{yy}^2 + \frac{1}{3}c_{12}^{(1)}S_{yy}S_{xx}^2 + \frac{1}{3}c_{22}^{(1)}S_{yy}^3 + \frac{1}{3}c_{66}^{(1)}S_{xy}^3 \\
&+ \frac{1}{4}c_{11}^{(21)}S_{xx}^4 + \frac{1}{2}c_{12}^{(21)}S_{xx}^2S_{yy}^2 + \frac{1}{4}c_{22}^{(21)}S_{yy}^4 + \frac{1}{4}c_{66}^{(21)}S_{xy}^4 + \frac{1}{4}c_{12}^{(22)}S_{xx}S_{yy}^3 + \frac{1}{4}c_{12}^{(22)}S_{yy}S_{xx}^3 \\
&- \frac{1}{2}\gamma_{31}^{(11)}S_{xx}^2E_z - \frac{1}{2}\gamma_{32}^{(11)}S_{yy}^2E_z - \frac{1}{2}\gamma_{31}^{(12)}S_{xx}E_z^2 - \frac{1}{2}\gamma_{32}^{(12)}S_{yy}E_z^2 \\
&- \frac{1}{3}\gamma_{31}^{(21)}S_{xx}^3E_z - \frac{1}{3}\gamma_{32}^{(21)}S_{yy}^3E_z - \frac{1}{2}\gamma_{31}^{(22)}S_{xx}^2E_z^2 - \frac{1}{2}\gamma_{32}^{(22)}S_{yy}^2E_z^2 \\
&- \frac{1}{3}\gamma_{31}^{(23)}S_{xx}E_z^3 - \frac{1}{3}\gamma_{32}^{(23)}S_{yy}E_z^3 - \frac{1}{3}\nu_1E_z^3 - \frac{1}{4}\nu_{22}E_z^4
\end{aligned} \tag{2}$$