Commun Nonlinear Sci Numer Simulat 16 (2011) 2936–2944

Contents lists available at ScienceDirect

Commun Nonlinear Sci Numer Simulat

journal homepage: www.elsevier.com/locate/cnsns

The parametric synchronization scheme of chaotic system

Zhenbo Li *, Xiaoshan Zhao

Department of Mathematics and Physics, Tianjin University of Technology and Education, Tianjin 300222, PR China

ARTICLE INFO

Article history:
Received 2 August 2010
Received in revised form 7 October 2010
Accepted 15 October 2010
Available online 23 October 2010

Keywords:
Parametric synchronization
Controller design
Hyperchaotic Rössler system

ABSTRACT

By constructing the parametric error vectors between drive system and response system, a parametric synchronization scheme of chaotic system which is different from all other schemes is proposed in this paper. Controller of the scheme is designed. The proposed scheme and controller not only realize the synchronization of the state vectors, but also synchronize the unknown response parameters to the given drive parameter as time goes to infinity. That is to say, to achieving the synchronization, we have no need to know the parameters of response system when the parameters of drive system are given. The scheme and controller are successfully applied to the Rössler and the hyperchaotic Rössler systems, corresponding numerical simulations are presented to show the validity of the proposed synchronization scheme and effectiveness of the controller.

1. Introduction

In this few decades, chaos study has increasingly become an important topic in nonlinear area. Chaos synchronization has also got the attention of researchers due to its potential application [1–5] to physics, secure communication, informatics, iatrology, etc. Since Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions [6] in 1990, a wide variety of approaches have been proposed for the synchronization of chaotic systems with impulsive control method [7], adaptive design method [8–12], backstepping design technique [13,14], active control [15,16] and so on. Meanwhile, many synchronization schemes have been proposed [17–23] such as complete synchronization, phase synchronization, anti-synchronization, lag synchronization, Q-S synchronization, generalized synchronization and projective synchronization, etc. However, none of the above-mentioned schemes consider constructing a parametric error vector between drive system and response system and realize the parametric synchronization.

In this paper, we propose a parametric synchronization scheme of chaotic system. The controller of this synchronization scheme is also designed. From theoretic analysis, we prove the validity of the scheme. Then the investigation of Rössler [24] and hyperchaotic Rössler [25] systems show the feasibility. We organize our paper as follows. The parametric synchronization scheme and the designing of its controller are presented in Section 2. In Sections 3 and 4, the parametric synchronization of Rössler system and hyperchaotic Rössler system are investigated. Corresponding numerical simulations are presented at the end of Sections 3 and 4. Finally the conclusions are drawn in Section 5.

2. Parametric synchronization scheme and its controller

Consider a class of nonlinear chaotic system described by

$$\dot{x} = f(x, a) \cdot x + C, \tag{1}$$

* Corresponding author. Tel./fax: +86 22 28116972.
E-mail address: lizhenbo126@126.com (Z. Li).

1007-5704/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.cnsns.2010.10.027

where $x \in R^n$ is an n-dimensional state vector of the system, $a = (a_1, a_2, \ldots, a_n)$ is the drive parameters vector, C is a constant matrix, $f(x, a_i)$ is a matrix function which contains the state variables and drive parameters. To study the parametric synchronization of system (1), we assume the response system as follows:

$$\dot{y} = f(y, \hat{a}) \cdot y + C + u, \tag{2}$$

where $y \in R^n$ is an n-dimensional state vector, $\hat{a} = (\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n)$ is unknown response parameters vector, C is a constant matrix, $u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$is the controller which would be designed later. $f(y, \hat{a}_i)$ is a matrix function which contains the state variables and unknown response parameters.

Define the parametric synchronization error vectors as

$$\begin{cases} e = y - x, \\ e_a = \hat{a} - a. \end{cases} \tag{3}$$

Then we can obtain the following error dynamical system by subtracting the drive system (1) from the response system (2)

$$(\dot{e}, \dot{e}_a)^{\mathrm{T}} = D(x, y, a_i) \cdot (e, e_a)^{\mathrm{T}} + U \tag{4}$$

where $e = (e_1, e_2, \ldots, e_n)^{\mathrm{T}}$ is error vector, $e_a = (e_{a_1}, e_{a_2}, \ldots, e_{a_n})^{\mathrm{T}}$ is the parametric error vector. $D(x, y, a_i)$ is the matrix function which contains the state variables and drive parameters. $U = (u_1, u_2, \ldots, u_n, u_{n+1}, u_{n+2}, \ldots, u_{2n})^{\mathrm{T}}$ is the controller.

Definition 1. For two systems described by system (1) and system (2), we say they are globally parametric synchronous if there exist a controller vector U such that

(1) All trajectories $x(t)$ in system (1) with any initial conditions approach the trajectories $y(t)$ in system (2) as time goes to infinity.
(2) The response parameters $\hat{a}_i(t)(i = 1, 2, \ldots, n)$ in system (2) with any initial conditions $\hat{a}_i(0)(i = 1, 2, \ldots, n)$ approach the drive parameters a in system (1) as time goes to infinity.

That is to say.

(i) $\lim_{t\to\infty} \|e(t)\| = \lim_{t\to\infty} \|y(t) - x(t)\| = 0.$
(ii) $\lim_{t\to\infty} \|e_a(t)\| = \lim_{t\to\infty} \|\hat{a}(t) - a\| = 0.$

which implies that the error dynamical system (4) between the drive system and response system is globally asymptotically stable.

Remark 1. A number of well-known chaotic systems can be described as system (1) and (2), such as Lorenz system, Rössler system, and hyperchaotic Rössler system, hyperchaotic Chen system, Chua's circuit etc.

Remark 2. The matrix function $D(x, y, a_i)$ in system (4) could not contain the unknown response parameters.

Remark 3. The update law of the unknown response parameters in system (2) can be obtained by solving the error system (4) under the controller U. More details will be seen in Sections 3 and 4. The drive parameters in system (1) have many available choices which could ensure the chaotic behavior of system (1).

Remark 4. The parametric synchronization scheme in this paper look similar to the synchronization of chaotic system with unknown parameters, but they are different. Our scheme constructs an error vector between the drive parameters and response parameters. And the response parameters, with arbitrary initial values, approach the drive parameters ultimately. That is to say, to achieving the synchronization, we have no need to know the parameters of response system when the parameters of drive system are given.

Then, we design the controller of the synchronization scheme. Let

$$D = D(x, y, a_i) = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1(n+n)} \\ d_{21} & & \cdots & \\ \vdots & \vdots & \ddots & \\ d_{(n+n)1} & & & d_{(n+n)(n+n)} \end{bmatrix}_{2n\times 2n},$$