Available online at www.sciencedirect.com
ScienceDirect
Communications in Nonlinear Science and Numerical Simulation 13 (2008) 707–714
Communications in Nonlinear Science and Numerical Simulation
www.elsevier.com/locate/cnsns

Fractional order adaptive high-gain controllers for a class of linear systems

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Received 6 June 2006; received in revised form 19 June 2006; accepted 19 June 2006
Available online 7 September 2006

Abstract

In this paper we show that a fractional adaptive controller based on high gain output feedback can always be found to stabilize any given linear, time-invariant, minimum phase, siso systems of relative degree one. We generalize the stability theorem of integer order controllers to the fractional order case, and we introduce a new tuning parameter for the performance behaviour of the controlled plant. A simulation example is given to illustrate the effectiveness of the proposed algorithm.

PACS: 05.45.−a; 02.30.Yy

Keywords: Adaptive high gain control; Fractional order control; Stability analysis

1. Introduction

Stability analysis and stability proof of fractional order control systems [5,19] have focused a great interest. This is due to the fact that the existing theory developed so far for stability proof mainly exists for integer order systems, and generally is not applicable to fractional order control systems. Many research works have tried to give answers to this question, using numerical or analytical arguments [2,6,10,14–16,18]. However the analytical proof of stability for fractional adaptive control schemes is up to now considered as an open problem [21]. The application of the theory of fractional calculus in adaptive control is just starting, but there are more and more works on this subject [11,12,20]. Recently, two of the present authors have presented a new scheme of fractional order adaptive $PI^{\lambda}D^{\mu}$ Controller [13], based on classical integer order algorithms [4]. They showed by simulations that the use of fractional order operators improves consistently the behaviour of the controlled plants, but the weakness of such work was the lack of theoretical arguments for guaranteeing the stability of such particular control schemes. All these works are based on the high gain output feedback control theory [1,3], which is very attractive because it is not based on system identification or plant parameter

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1007-5704/$ - see front matter
doi:10.1016/j.cnsns.2006.06.009

estimation algorithms or injection of probing signals. The main contribution of the present paper is to propose an analytical proof of stability for the generalization of this method to fractional order schemes in the case of linear minimum phase systems of relative degree one.

This paper is organized as follows:

In Section 2 mathematical definitions of fractional order derivative and integral are given, Section 3 presents the problem of fractional adaptive high gain controller stability. A theorem of stability is then proposed in Section 4 and its proof is detailed in Section 5. A simulation example illustrates this control method in Section 6. Section 7 presents the conclusion of the paper.

2. Mathematical aspects

The mathematical definition of fractional derivatives and integrals has been the subject of several different approaches [17]. In this paper we consider the Riemann–Liouville definition, in which the fractional order integrals are defined as

$$D_a^{-\mu} f(t) = \frac{1}{\Gamma(\mu)} \int_a^t (t-\xi)^{\mu-1} f(\xi) \mathrm{d}(\xi) \tag{1}$$

while the definition of fractional order derivatives is

$$D_a^{\mu} f(t) = \frac{\mathrm{d}}{\mathrm{d}t}[D_a^{-(1-\mu)} f(t)] = \frac{1}{\Gamma(1-\mu)} \frac{\mathrm{d}}{\mathrm{d}t} \int_a^t (t-\xi)^{-\mu} f(\xi) \mathrm{d}(\xi) \tag{2}$$

where

$$\Gamma(x) = \int_0^{\infty} y^{x-1} \mathrm{e}^{-y} \mathrm{d}y \tag{3}$$

is the Gamma function, $(a,t) \in \Re^2$ with $a < t$ and μ ($0 < \mu < 1$, $\mu \in \Re$) is the order of the operation.

For simplicity we will note $D^{\mu} f(t)$ for $D_0^{\mu} f(t)$.

3. Problem statement

Consider an uncertain siso system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4a}$$

$$y(t) = Cx(t) \tag{4b}$$

where $t \in \Re$ is the time variable, $x(t) \in \Re^n$ is the state with n unknown, $u(t) \in \Re$ is the scalar control and $y(t) \in \Re$ is the scalar measured output; A, B, C, are unknown matrices of appropriate dimensions. We assume the following.

Assumption A1. (A, B) is controllable and (C, A) is observable.

Suppose that (4a) is subject to a linear output feedback controller with gain $-k \in \Re$, i.e.,

$$u = -ky \tag{5}$$

Then the resulting feedback-controlled system can be described by

$$\dot{x} = \widetilde{A}(k)x \tag{6}$$

where

$$\widetilde{A}(k) \triangleq A - kBC \tag{7}$$

System (6) is asymptotically stable iff there exists $\epsilon > 0$ such that

$$Re(\lambda) \leqslant -\epsilon \quad \forall \lambda \in \sigma[\widetilde{A}(k)]$$

where $\sigma[\widetilde{A}(k)]$ denotes the set of eigenvalues of $\widetilde{A}(k)$. We introduce then the following definitions.