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On the various kinds of synchronization in delayed Duffing-Van der Pol system

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Abstract

A detailed investigation is performed about the various zones of stability for delayed Duffing-Van der Pol system, which in term shows the specific role of delay in the formation of the attractor. Coupling of two such similar systems gives rise to rich dynamics if the coupling also belongs to a delayed class. This gives rise to varieties of synchronization channels – such as anticipatory synchronization and retarded synchronization. As the coefficient of delayed term changes value, it is observed that anticipatory synchronization makes way to phase synchronization. The onset of these various mechanisms are tested by the computation of similarity function and probability of recurrence. In the study of phase synchronization the machinery of empirical mode decomposition (EMD) analysis is adapted and lastly maximal Lyapunov exponent is computed as a verifying criterion.

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1. Introduction

During the last decade chaotic synchronization [1] has become an area of active research [2] in various fields such as, nonlinear circuits, bio-physical systems, chemical reactions and so on. Synchronization phenomenon were demonstrated for electronic circuits [3], laser systems [4] and in secure communication processes [5]. The dynamics of the human cardio-respiratory system [6], of an extended ecological system [7],

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magnetocephalographic activity of Parkinson patients [8] and of electrosensitive cells of paddlefish [9] have been shown to display synchronization features.

There are different kinds of synchronization have been found: complete synchronization [10–12], generalized synchronization [12,13], phase synchronization [14–16], lag synchronization [17,18], anticipatory synchronization [19,20]. Of special interest is the synchronization of delay dynamical system, which is ubiquitous in nature and science because of finite speed of transmission. Among these anticipated and retarded [21,22] synchronization are two interesting phenomena in delay dynamical system. The discovery of anticipated synchronization [19] of chaotic system has led to extensive research in theoretical [19] and experimental [23,24] fields. In the study of anticipated synchronization, the self-time delay feedbacks are use either in driving system or driven system or both systems.

The phase synchronization phenomenon usually refers to the situation where the phases of the coupled system appears to have a certain relation but their amplitude remains chaotic and uncorrelated [15]. At first, phase synchronization was observed by Rosenblum et al. [14] in weakly coupled chaotic oscillators and their amplitudes were found to be weakly correlated. Later it was found that at strong coupling, the phase of two systems coincides and their amplitudes show a relatively high correlation. Again, when the phase difference of two systems are same, for certain time, then it is a case of phase locking. Phase synchronization of coupled chaotic oscillators has been demonstrated in electronic circuits, lasers, convective flow and other systems. It is very important in convective systems because, in planetary scale, it finds wide application in geophysical or atmospheric energetic transport.

It has been observed that chaotic phenomena observed in nonlinear dynamical systems can be well utilized for the communication of signal through synchronization. In fact the encryption of signals with the help of chaos is emerging as a new science. Among the various types of nonlinear systems exhibiting chaos, there exists a distinctive class of systems known as delay dynamical systems, useful in the domain of biophysics, feedback, circuits etc., which has a very special characteristics of being infinite dimensional. Such systems are now a days more often used for control and synchronization in practice. Among the various known nonlinear systems Duffing-Van der Pol occupies a central position, being the testing ground of many basic features of chaos theory.

Here in this communication, we have developed a theory for the delayed Duffing-Van der Pol system and its coupled form with a similar one, starting from the stability analysis up to the synchronization process. A novel feature of any delay dynamical system is the existence of more than one kinds of synchronization phenomena – the anticipatory or retarded synchronization. At the present moment it has been seen that such delayed dynamical finds exhaustive use in machine cutting, internet protocols, remote sensing problems. Except for the two synchronization procedure noted above which is usually termed as generalized synchronization, we have observed that in case such delayed systems there is a possibility of a transition from the generalized to phase synchronization depending upon the strength of coefficient of the delay term. This scenario bring us to the problem of identifying the phase of such a delayed system which is not an easy task as the corresponding phase space diagram do not exhibit a single point of rotation. As such the usual Hilbert transform or simplistic definition $\theta = \tan^{-1}(\frac{y}{x})$ does not work. So here we have to take recourse to the famous procedure of empirical mode decomposition proposed earlier by Huang et al. [25]. This procedure helps as to analyze the complicated quasiperiodic nature and the corresponding phase synchronization was detected through similarity function and probability of recurrence. In this connection it may be mentioned that the detailed analysis needed for the study of fixed points and their stability are also very much intricate due to the nonpolynomial structure of the eigen value equations involved. A proper formulation is needed here following earlier investigation which later forms the stepping stone of further study.

2. Formulation

2.1. Model

The system under consideration is a nonlinear oscillator governed by equation

$$\dot{X} = f(X),$$