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Hydrodynamic and thermal slip flow boundary layers over a flat plate with constant heat flux boundary condition

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ABSTRACT

In this paper the boundary layer flow over a flat plat with slip flow and constant heat flux surface condition is studied. Because the plate surface temperature varies along the x direction, the momentum and energy equations are coupled due to the presence of the temperature gradient along the plate surface. This coupling, which is due to the presence of the thermal jump term in Maxwell slip condition, renders the momentum and energy equations non-similar. As a preliminary study, this paper ignores this coupling due to thermal jump condition so that the self-similar nature of the equations is preserved. Even this fundamental problem for the case of a constant heat flux boundary condition has remained unexplored in the literature. It was therefore chosen for study in this paper. For the hydrodynamic boundary layer, velocity and shear stress distributions are presented for a range of values of the parameter characterizing the slip flow. This slip parameter is a function of the local Reynolds number, the local Knudsen number, and the tangential momentum accommodation coefficient representing the fraction of the molecules reflected diffusively at the surface. As the slip parameter increases, the slip velocity increases and the wall shear stress decreases. These results confirm the conclusions reached in other recent studies. The energy equation is solved to determine the temperature distribution in the thermal boundary layer for a range of values for both the slip parameter as well as the fluid Prandtl number. The increase in Prandtl number and/or the slip parameter reduces the dimensionless surface temperature. The actual surface temperature at any location of x is a function of the local Knudsen number, the local Reynolds number, the momentum accommodation coefficient, Prandtl number, other flow properties, and the applied heat flux.

1. Introduction

The boundary layer flow over a flat plate in a uniform stream of fluid has been studied extensively in fluid mechanics. When fluid flows in micro electro mechanical systems or MEMS are encountered, the no slip condition at the solid–fluid interface is abandoned in favor of a slip flow model which represents more accurately the non-equilibrium region near the interface. The first-order slip flow model proposes a relationship between the tangential component of the velocity at the surface, the velocity gradient normal to the surface, the mean free path, and the tangential momentum accommodation coefficient [1]. The effect of slip flow on the hydrodynamic boundary layer over a stationary flat plate has been studied by Martin and Boyd [2] who employed the Maxwell slip condition. The same problem has been studied by Vedantam [3] with three different models for the slip flow. The stationary plate model was extended by Fang and Lee [4] to a moving flat plate

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and by Anderson [5] to a stretching surface. For a stationary plate, the general conclusion from these investigations is that as the slip (rarefaction) parameter increases, the slip velocity increases and the wall shear stress decreases. For the moving plate, however, the momentum equation has two solutions and the location of the maximum shear stress which occurs on the plate surface for a stationary plate moves out into the fluid at a certain distance from the plate when the plate is moving. This distance depends on the ratio of the plate velocity and the free stream velocity [4].

The effect of slip flow on the flat plate thermal boundary layer characteristics has received limited attention. In a recent paper, Martin and Boyd [6] considered the thermal boundary layer over an isothermal flat plate in the presence of slip flow. In this analysis they took into account the effect of slip parameter K (a function of x, direction along the plate) in transforming the momentum equation and found that the stream function and consequently the y (direction normal to the plate) component of the velocity (v) also depends on the variation of the similarity variable f with respect to K. They neglected this in their earlier work [2].

With this modification, the similarity equation for the hydrodynamic boundary layer became a partial differential equation instead of the classical Blasius ordinary differential equation. With the introduction of temperature jump condition, the transformed energy equation also became a partial differential equation. They solved both equations numerically using center-difference approximation. With the neglect of temperature jump condition, the heat transfer was found to increase as the slip velocity increased. However, when the temperature jump condition was retained the heat transfer decreased as the slip velocity increased.

Other papers that are relevant to the present work are those of Wang [7], Anderson [8]. Abel and Mahesha [9,10]. Wang [7] considered the analysis of viscous flow due to stretching sheet with surface slip and suction, while Anderson [8] provided an exact solution of the Navier–Stokes equations describing the flow past a stretching boundary with partial slip. The papers by Abel and Mahesha [9,10] are important contributions because they included effect of variable thermal conductivity, heat source, radiation, buoyancy, magneto-hydrodynamic effects, and viscoelastic behavior of the fluid.

This paper considers the effect of slip flow on the thermal boundary layer over a flat plate with a constant heat flux boundary condition instead of a constant temperature boundary condition used by Martin and Boyd [6]. This situation arises in a MEM condensation application where a fixed heat dissipation (constant heat flux) due to condensation on lower surface of the plate is be removed by the gas flowing over the top surface. As a preliminary study the temperature jump condition will be neglected. As noted in [6], this is a reasonable assumption for the flow of liquids at the micro scale level particularly because of the lack of data on the thermal accommodation coefficient.

2. Momentum and energy equations

The continuity and momentum equations for the laminar hydrodynamic boundary layer over a flat plate in a uniform stream of fluid can be reduced to the Blasius differential equation.

$$2f''' + ff'' = 0 \tag{1}$$

where the dependent variable $f(\eta)$ and the independent variable η are defined as

$$f(\eta) = \frac{\psi(x,y)}{U_\infty\sqrt{vx/U_\infty}} \tag{2}$$

$$\eta = y\left(\frac{U_\infty}{vx}\right)^{1/2} \tag{3}$$

In Eq. (2), $\psi(x,y)$ is the stream function, U_∞ is the free stream velocity, x is the coordinate along the plate, y is the coordinate normal to the plate, and v is the kinematic viscosity of the fluid. The x component of the velocity, $u(x,y)$ is given by

$$f'(\eta) = \frac{u(x,y)}{U_\infty} \tag{4}$$

and the y component of the velocity, $v(x,y)$, is given by

$$v(x,y) = \frac{1}{2}\sqrt{vU\infty/x}(\eta f' - f) \tag{5}$$

The boundary condition at large normal distance from the plate is

$$f'(\infty) = 1 \tag{6}$$

For rarefied flow, the slip condition of the form given Martin and Boyd [2] is used.

$$u_{y=0} = \frac{(2-\sigma)}{\sigma}\lambda\frac{\partial u}{\partial y}\bigg|_{y=0} \tag{7}$$

where λ is the mean free path and σ is the tangential momentum accommodation coefficient. Eq. (7) may be expressed in dimensionless form as