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Efficient Computational Method for the Non-Probabilistic Reliability of Linear Structural Systems**

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ABSTRACT The non-probabilistic reliability in higher dimensional situations cannot be calculated efficiently using traditional methods, which either require a large amount of calculation or cause significant error. In this study, an efficient computational method is proposed for the calculation of non-probabilistic reliability based on the volume ratio theory, specifically for linear structural systems. The common expression for non-probabilistic reliability is obtained through formula derivation with the amount of computation considerably reduced. The compatibility between non-probabilistic and probabilistic safety measures is demonstrated through the Monte Carlo simulation. The high efficiency of the presented method is verified by several numerical examples.

KEY WORDS non-probabilistic reliability, linear structural system, formula derivation, compatibility, high efficiency

I. Introduction

The reliability analysis of practical engineering structures is closely related to uncertainties, which are inevitable when analyzing and designing structures[1–4]. The two main traditional methods for handling uncertainty, namely, probabilistic analysis and fuzzy-set-based theory, require sufficient information to determine the probability distribution or membership function[1–4]. However, experimental data are often inadequate in actual projects[5–9]. Compared with probability densities or membership functions, the approximations of the bounds of uncertain information are easier to gain[10]. The concept of the non-probabilistic safety of structures was first proposed by Ben-Haim[6] based on a convex model. He argued that the system was reliable if the accuracy of uncertainty was allowed to fluctuate within a certain range. However, he did not provide a specific non-probabilistic safety measure. Elishakoff[7] proposed the notion of non-probabilistic safety factor in the same year. For general cases where stress and strength were both interval variables, the non-probabilistic safety factor was defined as the ratio of the lower bound of strength to the upper bound of stress.

The theory of non-probabilistic reliability has been greatly developed in recent years. Guo[8] quantified uncertain structural parameters as interval variables and considered η, the shortest distance from the origin to the failure surface, as a measure of non-probabilistic reliability. The quantification was conducted by comparing the intervals of structural stress and strength. He defined that the structure was safe

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when $\eta > 1$ and dangerous when $\eta < -1$. The inequalities $\eta > 1$ and $\eta < -1$ denoted that the set of stress shared no common points with the set of strength, whereas the inequality $-1 < \eta < 1$ implied the interference of the two sets. This interference is the main focus of the current study. Sun[11] developed the possibility degree method for structure interval reliability analysis, which defined and formulated the structural safety possibility degree and the failure possibility degree for an interference model with two variables. A possibility degree method with good mathematical characteristics for analyzing structural interval reliability has been proposed and can be used regardless of the availability of sufficient probability information. Wang[9] regarded the volume ratio of the safe region to the total region constructed using basic interval variables as another measure of structural non-probabilistic reliability. This association was based on the non-probabilistic set-based stress-strength interference model established in his study. This kind of measure provided a clearer definition than the existing non-probabilistic measures. Hong[10] established a non-probabilistic model for structural reliability based on tolerance analysis and proposed a new measurement method of structural non-probabilistic reliability. The algorithm for multiple linear structures was developed and extended for multiple non-linear structures. Besides, some other non-probabilistic models have been proposed[12–17] and the applications in the actual engineering have been highly developed[18–20].

Furthermore, in the past years, researchers have studied the hybrid reliability analysis structures. When the probabilistic and interval variables appear in the same problem, numerical methods have been proposed. Wang et al.[21] developed a new hybrid reliability analysis technique based on the convex modeling theory for structures with multi-source uncertainties, such as randomness, fuzziness, and non-probabilistic boundedness. Based on the multidimensional parallelepiped convex model, a new method was proposed by Jiang et al.[22] for non-probabilistic structural reliability analysis, in which scattering levels for the parameters were expressed using marginal intervals, and the correlations between uncertain variables were expressed using relevant angles. Peng et al.[23] conducted the reliability analysis and optimization of laminates containing probabilistic uncertain variables and non-probabilistic bounded uncertain variables. Besides, the function approximation technique[24], the iterative rescaling method[25], the probability bounds approach[26], the mixed perturbation Monte Carlo method[27] and the complex nesting optimization algorithm[28] have also been created to tackle the hybrid problem. All in all, the hybrid reliability analysis is still in its preliminary stage. The most common way to deal with the issue is the two-layer nesting optimization, in which one layer is for probability reliability analysis in terms of random variables and the other for non-probabilistic analysis in terms of interval variables. Therefore, in order to reduce the amount of computation and improve the computational efficiency, the new computational methods for interval variables with high-performance are always necessary in either non-probabilistic reliability analysis or hybrid reliability analysis.

In this paper, the ranges of structural stress and strength are described by intervals. First, various non-probabilistic reliability models for structures are summarized, and the difficulty of computing non-probabilistic reliability efficiently in higher dimensional spaces is discussed. Then, when the performance function of the structure is linear, an accurate solution for the non-probabilistic reliability model of the structure is obtained and verified through the formula derivation, i.e. mathematical induction. Next, the compatibility between the presented non-probabilistic safety measure and the probability reliability based on the probability theory is demonstrated through the Monte Carlo simulation. After that, two numerical examples are provided to demonstrate the validity and high efficiency of the proposed method. Finally, some conclusions are summarized.

II. Reliability Model based on Non-Probabilistic Set Theory

In practical engineering applications, stress S and strength R can be expressed as follows:

$$S = S(s_1, s_2, \cdots, s_m) \tag{1}$$

$$R = R(r_1, r_2, \cdots, r_n) \tag{2}$$

where s_i $(i = 1, 2, \cdots, m)$ refers to the factors that influence structural stress, such as force, moment, humidity, temperature and overloading; r_j $(j = 1, 2, \cdots, n)$ refers to the factors that influence structural strength, such as material property and crack length.